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Capital, Energy and Labour Substitution: The Supply Block in OECD Medium-Term Models

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by

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This paper sets out the various stages in the construction and estimation of a supply block for medium-term projection models. It describes the theoretical basis for the specification chosen and the estimation results.

The basic block is a set of production factor demand functions. It is refined by modelling the scrapping behaviour of firms, introducing the effects stemming from movements in hours worked, and examining the role of profits. Other possible changes or improvements are outlined. The model is used to simulate the effects of movements in the relative prices of labour, capital and energy in recent years.

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CAPITAL, ENERGY AND LABOUR SUBSTITUTION: THE SUPPLY BLOCK IN OECD MEDIUM-TERM MODELS

I. Introduction

1. To construct medium-term projection models it is necessary to estimate production functions. There is considerable scope in the choice of function: number of factors, specification, separability, functional form, etc. The first point to be tackled is the extent of factor substitutability; should the function be clay-clay, i.e. with no possibility of substitution between factors, putty-putty, with substitutability at the time of purchase and also throughout working life, or putty-clay, with substitutability only at the time of purchase of equipment, and with the factor proportions thereafter remaining fixed throughout its working life?

2. Two difficulties were experienced in estimating clay-clay functions: the marginal employment and energy series, i.e. the quantities allotted to each vintage, implicitly implied by the estimation were found to be highly erratic. Also, and more seriously, while the results appear accurate when the period of estimation ends before the first oil crisis, they are severely disturbed when more recent years are included. This probably indicates that movements in relative prices of labour, capital and energy affect the choice of production techniques and productivity trends, and so recourse must be had to production functions which allow for factor substitution.

3. The estimation of putty-putty production functions indicated very long adjustment lags for capital. Various tests were carried out on factor adjustment following changes in relative prices and they gave some indication that conditions for most countries were closer to putty-clay than to putty-putty technology. These tests are based on the notion that in the first case the level of relative prices influences investment, while in the second case changes in relative prices are influential since the factor proportions used with existing equipment have to be reassessed. However, the tests did not appear to be conclusive.

4. A production function of the putty-clay type was adopted, with three factors of production. It appears essential to include energy if the aim is to explain the breaks observed in series in the period around 1973-74. It was finally decided that a fourth factor, non-energy raw materials, would not be included on account of the difficulty of obtaining reliable statistical series and the very great complexity of estimation its inclusion would have caused.

5. The choice of a putty-clay technology has the added advantage of enabling short-term productive potential to be clearly defined: it is a quantity that can be produced with the equipment installed, each item of equipment having a fixed technology. With a putty-putty function, it is always possible to produce more with a given amount of capital by increasing the quantities of energy or labour utilized.

6. The functional form of the <u>ex ante</u> production function has to satisfy several criteria. It should be relatively easy to handle, not impose any unduly heavy <u>a priori</u> constraint on the elasticities of factor substitution, and allow easy calculation of productive capacity. The latter is used at many points in macro-economic models, notably in the equations for prices and foreign trade. The last criterion ruled out the use of cost functions, e.g. translog, since although they were very simple to handle and very general, they were not easy to introduce in a macro-economic model. A "nested CES" type of function was finally chosen, in which a CES function for the first factor was combined with a CES function for the other two factors. The advantage of this function is that it does not pre-judge the two factors contained in the nested (inner) CES function to be all-in-all substitutable or complementary since the result depends on the estimated values of both substitution elasticities.

7. This type of function poses estimation difficulties. The direct approach, which consists in explaining observed output, is very difficult, firstly on account of collinearities and secondly, because the quantity of factors available will enable more to be produced than actually is produced. Indicators of intensity of utilisation for all factors are therefore needed, and these generally do not exist. The approach decided upon was to estimate factor demand functions assuming a certain type of behaviour by firms. If they minimize their expected output cost and are subject to a market constraint, factor demands will depend on relative factor prices. If, on the other hand, they maximize their profit, they will themselves set their own level of output and factor demands will depend only on real factor prices relative to the output price.

8. It is possible to choose between these two types of firms' behaviour by analysing the manner in which output prices are determined(1). It was possible to reject, for all countries except Japan, the assumption that firms maximize their profit. Japan being an ambiguous case, the assumption adopted for all the countries was that expected cost would be minimized.

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(1) Details are available on request.

II. The Putty-clay production function and the model estimated

9. After choosing the type of technology and the functional form, several points remain to be specified. First, the three factors need to be allocated within the two CES functions. If, for example, the production function is written:

$$Q = f(g(K,E),L)$$

where Q = output
K = capital
E = energy
L = labour

and where we write $Z = g(K,E)$

and labour being weakly separable, a calculation shows that the partial elasticities of substitution between capital and labour and between energy and labour are equal. The reason is that in this case, the cost function associated with the production function shows the same weak separability as the latter, and if this function is denoted by C:

$$\sigma_{KL} = \frac{C.C_{KL}}{C_K C_L} = \frac{C.C_{WL}}{C_W C_L} = \sigma_{EL}$$

10. To verify the specification a translog type of cost function was estimated. This allows considerable freedom for adjustment of elasticities of substitution. The estimation showed that the assumption of weak separability of labour was not rejected for any country, whereas in the case of Canada the assumption of separability of energy was rejected. The impossibility of treating the capital-labour "bundle", i.e. value added, as a single entity is confirmed by other studies, e.g. Berndt-Wood (1975). However, the tests carried out do not appear to have been very stringent, and varied results appear in the literature. For example, in their inter-country study, Gregory and Griffin (1976) conclude that value added can be separated. While the point remains debatable, in the remainder of this study the separability of labour is assumed and it is therefore linked by a CES function to another CES function aggregating capital and energy.

11. The second decision concerns scrapping. The "putty-clay" production function defines the possible choices of technology at the time of purchasing equipment, and therefore links investment and the quantities of labour and energy allocated to new equipment. Alongside its purchases of equipment, the firm will decide whether to keep or scrap existing equipment. Behaviour of the firm will be considered later on in this paper. For the moment, it will be assumed that each vintage of equipment is assigned the same constant

scrapping rate and therefore depreciates geometrically from the first period after it is installed. This assumption implies that employment and energy are also assigned this same rate(1). Throughout the first part of this paper, changes in weekly or yearly hours worked will also be ignored and they will implicitly be assumed to be constant. This assumption will be reconsidered later on.

12. In certain cases a simplification of this scheme was possible. When the elasticity of substitution between capital and labour appeared to be very close to unity, labour and the capital-energy bundle were related not by a CES function but by a Cobb-Douglas function. This has the advantage of yielding much simpler expressions for optimal factor demands. A simple two-factor CES function was therefore estimated first of all for capital and labour. In the case of France, Canada and the United States, the elasticity of capital-labour substitution obtained did not differ significantly from unity(2). Accordingly, a CES plus Cobb-Douglas type of function was finally adopted for these three countries. On the other hand, for Japan, Germany and especially the United Kingdom, the values obtained were less than unity(3) which justified maintaining the "nested CES" production function.

13. It is necessary to consider in greater detail the behaviour of firms. They are subject to a marginal production function written as follows:

- in the "nested CES" case:

(1) $$Q = Q_0 \, e^{rT} \left(Z^{\sigma_1 - 1/\sigma_1} + a_L \, L^{\sigma_1 - 1/\sigma_1} \right)^{\sigma_1/\sigma_1 - 1}$$

with (1') $$Z = \left(a_K \, K^{\sigma - 1/\sigma} + a_E \, E^{\sigma - 1/\sigma} \right)^{\sigma/\sigma - 1}$$

- in the "CES/Cobb-Douglas" case:

(2) $$Q = Q_0 \, e^{rT} \, Z^a \, L^{1-a}$$

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(1) It would have been very much more complicated at this stage to assume that the lifetime of equipment was exogenous and constant, since factor proportions vary from one vintage to another and so this would have implied different scrapping rates for the three factors, with the differentials depending on the whole historical trend of relative prices.

(2) 0.98 for France, 1.08 for the United States, 0.98 for Canada.

(3) 0.58, 0.65 and 0.31, respectively.

with Z still defined by equation (1')

where Q = potential output with the vintage of equipment installed

K = capital for that vintage

E = energy for that vintage

L = labour for that vintage

e^{rT} = trend in autonomous technical progress at rate r (T = time)

σ = elasticity of substitution between capital and energy

σ_1 = elasticity of substitution between the capital-energy bundle and labour

Q_0, a_K, a_E, a_L = constants

a = share of the capital-energy bundle in value output for the vintage (constant returns to scale in every case).

14. The definition of the Q variable requires comment. With only two factors (capital and labour), Q is value added in volume on a given vintage of equipment. If an energy factor is included, then the sum of value added and of intermediate energy consumption is used. Rigorously, the latter calculation is valid only if value added is computed in the national accounts as the difference between gross output, for which there are statistics, and all intermediate inputs. In the present case, an output series has been constructed, but with the difference that non-energy intermediate consumption is ignored.

15. Firms are assumed to minimize their costs. The model proposed by Ando-Modigliani-Rasche-Turnovsky (1974) and subsequently taken up by De Menil-Yohn (1977) and Maurice-Villa (1980) can therefore be adapted to the three-factor case. All the calculations leading to the expressions for optimal factor quantities as well as the expressions themselves, are given in Annex 1. From these expressions it is apparent under what conditions capital and energy are complementary or substitutable overall. In the case of the Cobb-Douglas plus CES function, the condition is simple. If $\sigma > 1-a$, i.e. if the elasticity of substitution is greater than the labour share in output, an increase in the energy price will stimulate investment which will therefore appear as substitutable for energy overall. In the "nested CES" case it would be necessary that $\sigma > \sigma_1$ be in the same

situation of overall substitutability (i.e. for the elasticity of capital/energy substitution to exceed that for labour/energy) though this condition while necessary is not sufficient(1).

16. That establishes how the firm fixes the optimum quantities for factor demands once its market expectation Q is arrived at. As regards the way in which this expectation will be specified, a very simple method used by Bischoff (1971) has been adopted. The firms have a desired output capacity related to past output levels:

$$X^* = \sum_i a_i X_{-i}$$

It will continue to be assumed that all vintages of equipment are scrapped at a single rate . The desired gross change in capacity is then equal to:

$$(3) \quad \Delta Q = \Delta X = X^* - (1-\delta)\, X^*_{-1} = \sum_i a_i \,(X_{-i} - (1-\delta) X_{-i-1})$$

and this expression is used to identify Q in equations (B) and (C)(2). After estimating equations (B) and (C) and constructing, as shown below, the capacity utilisation rate (TUC), this specification of the desired change in capacity can be refined and written exactly as the difference between expected output and existing capacity, i.e.:

$$(4) \quad \Delta X = X^* - (1-\delta)\, X_{-1}\, (\overline{TUC}/TUC)_{-1}$$

where $\overline{TUC}$ is the average value for the utilisation rate. A specification was also used in which firms only partly adjusted deviations in the utilisation rate from its normal value, i.e.:

$$(5) \quad X = X^* - (1-\delta)\, X_{-1}\, (\overline{TUC}/TUC)^{\mu}_{-1}$$

(5) also implies that investment changes as long as the rate of utilisation differs from its average value. France is the only country for which going from (3) to (5) enabled the equation to be improved. For all other countries the very simple construction (3) was adequate.

17. It would obviously be highly desirable to be able to estimate equations (B) or (C) simultaneously. However, to do so would require construction of series for L and E, employment and energy assigned to the latest vintage. An attempt was made to represent them by gross change (allowing for scrapping) in total employment and in total intermediate

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(1) In this case the derivative of K* (optimal investment) with respect to e (energy price) is complicated and there is no independent analytical condition on variables making it positive.

(2) See these equations for optimum factor demand in Annex 1.

energy consumption, but the results obtained for the estimation were strange for certain countries and not very stable. Results reported here are, therefore, derived from estimating the investment equation alone(1).

18. The model finally estimated comprises the investment equation (B1) or (C1), $\bar{Q}$ being defined by (3). This equation is estimated in logarithmic form, following King (1972), so reducing the number of non-linearities. Since relative price expectations are obviously not instantaneous, lag structures for the relative price terms are added. After adjustment of (B1) or (C1), values for the basic parameters r, a, σ, σ_1 were estimated(2). These values enable the *marginal* optimal quantities for labour and energy to be calculated using (B2) and (B3) or (C2) and (C3). These quantities are then combined, with allowance being made for the scrapping rate to obtain the *total* optimal quantities for labour and energy. This gives:

(6) $$EN^* = (1-\delta)\ EN^*_{-1} + E^*$$

(7) $$N^* = (1-\delta)\ N^*_{-1} + L^*$$

where EN^* and N^* = *total* optimal quantities for energy and labour

E^* and L^* = *marginal* optimal quantities calculated from (B2) and (B3) or (C2) and (C3) in Annex 1.

It should be noted that equations (6) and (7) define the optimal quantities of inputs required to achieve potential output, i.e., by utilising all unscrapped capital stock of the various vintages. If this capital is under-utilized, for example because demand is low, actual optimal employment and optimal energy are lower and they can then be calculated using:

(8A) $$EN^{**} = EN^* \,.\, TUC$$

(8B) $$N^{**} = N^* \,.\, TUC$$

where TUC represents the rate of utilisation of existing capacity.

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(1) This single equation approach precludes any study of inter-related adjustment of factors, as in Nadiri-Rosen (1968), Lucas (1967) or Schramm (1970).

(2) Here it should be pointed out that the investment equations are over-identified in the (B1) or (C1) form. They must be divided by $a_K\sigma$ for estimation. To facilitate estimation, the value of the constant $(a_E/a_K)\sigma$ was pre-assessed by estimating (B4) or (C4).

19. The capacity utilisation rate, too, is simply derived from the model. Potential output XPOT is the cumulative total of additions to capacity Q after allowing for scrapping:

(9) $$XPOT = XPOT_{-1}\,(1-\delta) + \bar{Q}$$

The capacity utilisation rate is the ratio of actual output to potential output:

(10) $$TUC = X/XPOT$$

It may be noted that, in this first simple version of the model in which the scrapping rate is fixed and exogenous, additions to capacity are defined by (3) and investment is determined by (B1) or (C1) without bringing in any additional variable (profits, etc.), the utilisation rate in the long term, always moves back towards its normal level. If demand - and therefore actual output - changes sharply, in the short term TUC will move, though gradually, with a lag depending on the a_i values in (3), and firms will adjust their output capacity accordingly.

20. The final forms now have to be given to the equations for total employment and total intermediate energy consumption. It was assumed that they would gradually adjust to their desired levels, allowing for the actual rate of capacity utilisation, which is written in the logarithmic form(1):

(11A) $$\log EN = \lambda_1 \log EN_{-1} + (1-\lambda_1) \log EN^{**}$$

(11B) $$\log N = \lambda_2 \log N_{-1} + (1-\lambda_2) \log N^{**}$$

The idea has sometimes been put forward that the elasticity of employment and energy with respect to the utilisation rate is not unity since the average age of the machinery used changes when demand fluctuates, with firms giving priority to the most up-to-date machinery. This can be expressed in the form:

(12A) $$\log EN = \lambda_1 \log EN_{-1} + (1-\lambda_1)\,[\log EN^{*} + b_1 \log TUC]$$

(12B) $$\log N = \lambda_2 \log N_{-1} + (1-\lambda_2)\,[\log N^{*} + b_2 \log TUC]$$

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(1) In the construction of the final form of the models, concurrently with (11B) and (12B), we estimated a process of adjustment of employment to desired employment derived from an error correction model [see Hendry, Von Ungern-Sternberg (1981)]. The employment equation is then written:

(12C) $$\log N - \log N_{-1} = \lambda_3\,(\log N^{*} + b_2 \log TUC - \log N^{*}_{-1} - b_2 \log TUC_{-1}) + \lambda_4\,(\log N^{*}_{-1} + b_2 \log TUC_{-1} - \log N_{-1})$$

This second type of equation performed well for all the countries.

Table 1

Estimation of the Investment Equation

(Annual data 1964-1979)	Type of function	r technical progress	Elasticities of substitution		a_i accelerator coefficients				R^2	DW	SEE (%)
			K/E	L/KE	a_o	a_1	a_2	a_3			
United States	CES-CD	-0.005 (0.9)	0.26 (1.4)	1	0.24 (10.0)	0.30	0.28 (23.3)	0.18 (15.0)	0.96	2.19	3.3
Germany	CES-CES	0.011 (5.8)	0.60 (3.6)	0.50*	0.33 (16.5)	0.30	0.23 (22.9)	0.14 (14.0)	0.98	2.28	2.8
France	CES-CD	0.029 (1.7)	0.37 (1.8)	1	0.40 (4.9)	0.37 (4.4)	0.23		0.97	1.04	5.2
United Kingdom	CES-CES	-0.004 (1.5)	0.41 (1.2)	0.20*	0.16 (4.4)	0.30	0.32 (17.8)	0.22 (12.2)	0.94	1.58	3.3
Japan	CES-CES	0.020 (3.4)	0.28 (4.5)	0.50*	0.18 (5.0)	0.30	0.31 (17.2)	0.21 (11.7)	0.98	1.94	3.5
Canada	CES-CD	0.019 (1.8)	0.57 (4.7)	1	0.23 (3.7)	0.39 (6.7)	0.38		0.97	1.24	5.0

* Obtained by iteration.

21. The results for the investment equation are given in Table 1. The explained variable is total productive investment by firms (equipment and buildings), excluding the farm sector(1) and excluding the energy sector for Canada. The explanatory variables are value-added in the sector of the investment, cost of capital use - constructed as shown in Annex 1, from the rate of interest on bonds and the investment price - per head wage costs (ratio of wages and social insurance to number of employees in the sector of the investment), and the price of intermediate energy consumption, reconstituted from the wholesale price indices for the countries for which national accounts series are not available. In the case of nested-CES production functions, it was necessary to proceed by iteration for the elasticity of substitution between labour and the capital-energy bundle since direct estimation was found to be very difficult. The elasticities of substitution obtained imply that, on the whole, investment and energy are only substitutable in Germany and the United Kingdom, and these are countries for which a particularly low value is obtained for the elasticity of substitution with labour. In all the other countries, a rise in the energy price reduces investment, and the predominant effect is the substitution of labour for the other two factors(2).

22. The equations for employment and energy(3) are given in Table 2. Where employment is concerned, elasticity with respect to the utilisation rate is usually unity. On the

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(1) Except for Japan.

(2) The energy demand price elasticities resulting from these estimations are on average:

United States:	-0.28	United Kingdom:	-0.35
Germany:	-0.64	Japan:	-0.30
France:	-0.49	Canada:	-0.60

They are fairly high, particularly in Germany, France and Canada, although it should be remembered that these are long-term elasticities since they apply to overall energy consumption only when all equipment has been renewed. For a five-year period, for example between 1973 and 1978 after the first oil shock, roughly one-half of all vintages of equipment was replaced, and the apparent price elasticities of intermediate energy consumption are also one-half of those given above.

(3) Total employment (employees plus self-employed) in the sector of the investment, and intermediate energy consumption by firms, were reconstituted country by country from various sources: national accounts, national energy statistics, and IEA energy balances.

Table 2

Estimation of the Employment and Energy Equations

(Annual data 1964-1978)	Log. of the "optimal" variable	Constant	Log. of the utilisation rate	R^2	DW	SEE (%)
United States						
employment	0.58 (9.5)	0.005 (5.4)	1	0.99	1.54	0.8
energy	0.65 (5.4)	0.15 (2.8)	1.6 (4.0)	0.96	1.27	3.3
Germany						
employment	0.41 (11.8)	0.007 (0.2)	1.53 (2.8)	0.98	1.35	0.5
energy	0	-0.012 (0.7)	1.37 (7.8)	0.99	2.05	2.1
France						
employment	0.77 (10.1)	0.011 (0.5)	1	0.98	1.04	1.0
energy	0.21 (3.5)	0.040 (2.3)	1.98 (9.0)	0.99	1.03	1.8
United Kingdom						
employment	0.71 (24.6)	0.100 (3.8)	1.44 (7.6)	0.97	1.77	0.5
energy	0.48 (4.3)	0.206 (1.8)	1.81 (2.4)	0.87	1.22	3.5
Japan						
employment	0.83 (16.0)	0.084 (2.2)	1	0.99	2.56	0.9
energy	0.29 (3.0)	0.176 (1.9)	2.20 (3.0)	0.89	1.33	2.8
Canada						
employment	0.74 (7.7)	0.045 (1.0)	1	0.99	1.63	1.3
energy	0.58 (8.5)	0.062 (1.3)	1.31 (4.6)	0.99	2.37	2.6

other hand, that elasticity is always greater than unity for energy, which could indicate that, in output surges, more heavily energy-consuming equipment is brought into operation. Employment adjustment is fairly rapid in Germany and the United States (with an average adjustment lag of about one year), fairly slow in France, the United Kingdom and Canada (a lag of about three years), and very slow in Japan (average lag of nearly five years). Energy consumption adjusts very quickly, except in Canada and the United States where the average lag exceeds one year.

III. Modelling of scrapping

23. The equations arrived at in Part II were constructed on the assumption of an exogenous and constant scrapping rate for each vintage of equipment. Since it is possible that output prices do not cover output costs with certain equipment this assumption should be relaxed. In this event, the traditional theory of scrapping states that firms get rid of such equipment. Another possibility is that as soon as the cost of installing and operating new equipment becomes lower than the operating cost of old equipment, the latter is scrapped. It is therefore important to examine the relationships between scrapping and either the ratio of output price to output cost or the ratio of development cost (cost of bringing new equipment into operation) to output cost.

24. The way scrapping is modelled obviously depends on the way price determination by firms is represented. If it was assumed that firms were maximising their long-term profits, output prices would be equal to the development cost and both conceptions of scrapping outlined above would amount to the same thing. If pricing were to be represented as a strict maximisation of short-term profit, the prices would therefore be equal to the unit cost of output with the least efficient equipment required to satisfy demand (i.e. equal to the short-term marginal cost), and firms would never have any problem of profitability on existing capacity. In the medium-term models in which these supply blocks are located, prices are based on the average unit cost of output, which makes it possible to consider scrapping from either of the two viewpoints.

25. There are no statistical series available on scrapping, and this cannot therefore be modelled directly. It was captured through movements in investment(1), hence through fluctuations in replacement investment. This undoubtedly introduces a bias in the results since firms may dispose of unprofitable equipment without replacing it. Moreover, no model has been built in which the date of scrapping of each vintage is explicitly represented. It seemed far too complicated to estimate both the parameters for the ex ante production function and a vintage-by-vintage scrapping

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(1) And not through the market value of firms, as suggested by Baily (1981).

behaviour pattern. We have therefore simply tried to model the apparent average scrapping rate for all existing equipment. This means that a single scrapping rate can be used as before for the three factors of production.

26. The scrapping rate δ is defined by:

$$(13) \quad \delta_t = Q_{t-T} / \sum_{\theta=0}^{T} Q_{t-\theta}$$

where Q_j is the capacity installed at date j and T is the age of the equipment scrapped in the period t. In order to establish the formulae for estimating δ, we have made considerable simplifications. In the more general case, it is impossible to write an estimable analytical form for δ. If the first scrapping assumption is reverted to and it is assumed that a piece of equipment is scrapped when the variable costs exceed the value of the output which can be obtained using it, and if it is assumed that the situation at the outset is one of steady-state growth, a usable expression for δ is obtained.

27. After simplification and linearisation, we arrive at(1):

$$(14) \quad \delta = A + B/(1-P_V V/PQ) + C(P_V \dot{V}/PQ) \qquad B, C > 0$$

where $P_V V/PQ$ stands for the share of cost relating to the variable factors in the value of output. According to the other possible version of the scrapping condition, firms do not scrap when the vintage concerned becomes unprofitable, but when the total discounted cost of a new item of equipment becomes lower than the variable cost with old equipment. This amounts to testing:

$$(15) \quad \delta = A + B/(1-P_V V/PdQ) + C\,(P_V \dot{V}/PdQ) \qquad B, C > 0$$

Pd is the development cost, equivalent to the discounted output cost (at the optimum) with new equipment(2).

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(1) For details of the calculation, see Annex 2.

(2) In the case of CES + Cobb-Douglas production function, Pd is written:

$$(16A) \quad Pd = P_0 w^{1-a}(a_K^{\sigma} u^{1-\sigma} + a_E^{\sigma} e^{1-\sigma})^{a/1-\sigma} e^{-rT}$$

with the same notation as in Part II and Annex 1.

Where the "nested CES" function is used:

$$(16B) \quad Pd = P_0(P_{KE}^{1-\sigma_1} + a_L^{\sigma_1} w^{1-\sigma_1})^{1/1-\sigma_1} e^{-rT}$$

where $P_{KE} = (a_K u^{1-\sigma} + a_E^{\sigma} e^{1-\sigma})^{1/1-\sigma}.$

28. In order to estimate the parameters of the two possible formulations for the scrapping rate, (14) and (15), an expression for desired change in capacity (equation (3)) in which the scrapping rate is expressed as in (14) or (15) was introduced in the investment equations ((B1) or (C1) in Annex 1). We then estimate the parameters for both the investment function and the scrapping rate. Equations (14) and (15) have been complicated by introducing the possibility of lags for the variable cost share and its rate of growth, since firms undoubtedly do not respond instantaneously to a change in relative prices but wait for it to be confirmed. Having obtained the parameters contained in the expression for δ, optimum employment, optimum intermediate energy consumption and potential output can be constructed as before using the estimated flexible scrapping rate.

29. The results obtained are summarized in Table 3. For all countries except the United Kingdom, the best results are given by the ratio of variable costs to output and not to development costs: for these countries, scrapping apparently occurs when profits become insufficient and not when it becomes profitable to purchase new equipment. For Japan and Canada it is found that the parameterisation of δ is highly significant. It is almost significant for Germany and France and of very low significance for the United States and the United Kingdom.

30. If these estimations were reliable, it would be possible to rank countries according to the intensity with which their industrial base has been modernized following increases in costs. In all cases, with the exception of Japan and Canada, introducing the parameterisation for the scrapping rate reduces the estimated elasticity of substitution between capital and energy. This is normal. If, following an increase in the energy price, energy consumption is reduced, and if the scrapping rate is fixed, this reduction will be entirely attributed to *ex ante* substitution. If the rate is flexible, the reduction will be attributed partly to *ex ante* substitution, partly to the scrapping of equipment for which energy productivity is lowest.

31. The estimates selected for the endogenous scrapping rate are therefore (for all countries, even those for which the significance of parameterisation is doubtful):

United States: $\delta = 0.083 + 0.017\ (1/1-P_V V/PQ)$ (maximum lag: one year)

United Kingdom: $\delta = 0.10 + 0.055\ (P_V \dot{V}/PdQ)$ (maximum lag: one year)

Germany: $\delta = 0.006 + 0.066\ (1/1-P_V V/PQ)$ (maximum lag: two years)

Table 3

Estimation of the Investment Function with Endogenous Scrapping Rate

(Annual data, 1964-1978)	r technical progress	Elasticities of substitution		Endogenisation of δ						R^2	DW	SEE (%)
				Output prices (14)			Development cost (15)					
		K/E	L/KE	A	B	C	A	B	C			
United States	-0.002 (0.5)	0.06 (0.5)	1*	0.083 (1.5)	0.017 (1.1)					0.98	2.34	2.7
Germany	0.020 (6.6)	0.23 (1.7)	0.80**	-0.006 (0.1)	0.006 (1.6)					0.99	2.37	2.1
France	0.026 (5.8)	0.27 (2.2)	1*	0.09 (0.9)		0.17 (1.8)				0.98	1.55	4.6
United Kingdom	-0.006 (2.2)	0.36 (2.1)	0.15**				0.10 (0.7)		0.055 (0.9)	0.95	1.32	3.3
Japan	0.003 (0.7)	0.40 (4.5)	0.30**	0.059 (9.8)		0.19 (4.5)				0.99	2.22	2.3
Canada	0.053 (6.2)	0.57 (5.2)	1*	-0.03 (1.2)	0.015 (5.4)					0.99	2.01	2.6

*: Cobb-Douglas function.

**: Obtained by iteration.

France: $\delta = 0.090 + 0.17\ (P_V\dot{V}/PQ)$ (maximum lag: two years)

Japan: $\delta = 0.059 + 0.19\ (P_V\dot{V}/PQ)$ (maximum lag: one year)

Canada: $\delta = 0.03 + 0.015\ (1/1-P_VV/PQ)$ (maximum lag: one year)

32. It will be noted that it was in no case possible to obtain simultaneously significant values for the level and the growth rate of the share of costs; in addition to the foregoing differences between the countries, there might therefore be a difference between those where the increase in costs entails extra scrapping that is permanent and those where the scrapping entailed is only temporary (the United Kingdom, France and Japan are in the second category). In order to judge these estimates, the following variance effect was calculated: according to the estimates what would be the consequences for the average lifetime of equipment of a one-point increase (in 1979) in the variable costs share in output (valued at the development price for the United Kingdom)? These estimated consequences were compared to those derived applying the usual strict scrapping condition(1). The results are shown in the table below:

Effect on Average Lifetime
of a 1-point Increase in the Cost Share

Unit: year	Short-term	Long-term	Theoretical*	Estimated/ theoretical
United States	-0.10	-0.10	-0.90	11%
United Kingdom	-0.01	0	-0.56	2%
Germany	-0.17	-0.17	-0.43	40%
France	-0.03	0	-0.51	6%
Japan	-0.06	0	-0.30	20%
Canada	-0.59	-0.59	-0.89	66%

* i.e. strict application of the usual scrapping condition.

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(1) In the latter case the increase in cost was compared to the ratio of trend labour productivity growth in the various countries.

33. Those countries for which only the growth rate of the cost share is significant obviously show no long-term effect. The short term here, however, means after the estimated time-lag for the effect to work through to the scrapping rate. It was observed that for all countries scrapping was less drastic than the strict theoretical condition implies. Firms may hesitate to write off equipment definitively even if it is unprofitable, since the act is irreversible. It was for Canada and Germany (and to a lesser extent Japan) that estimated scrapping was closest to theoretical scrapping, and therefore perhaps that the modernisation effort has been most intense. For the three other countries, scrapping showed little sensitivity (and even none at all in the United Kingdom) to the trend in costs.

IV. Hours worked

34. All earlier estimates disregarded the problem of how many hours were being worked, weekly or annually, making no distinction between numbers employed and hours worked. This aspect is now addressed, first by constructing hours-worked equations and then by considering how changes in the number of hours worked intervene in the supply block in determining investment, numbers employed, etc.

35. Annex 3 contains a theoretical model for the way in which a firm apportions its labour input as between employment and hours worked. Hours worked are not treated as a fourth factor. Proceeding hierarchically, employment and hours worked were apportioned after determining the quantity of labour service required in the three-factor production function. The result of this model was that hours worked deviated from exogenous trend hours worked when the capacity utilisation rate or output growth rate deviated from their mean value; moreover, those deviations had an asymmetrical effect depending on whether they were positive or negative. Again, hours worked were affected positively by the relationship between the employment adjustment cost and the hourly wage cost. This relationship is hard to measure rigorously, but it was approximated by using the series available for the share of labour cost not directly associated with wages (cost of hiring, training, holidays, safety, etc.) in total wage cost, which provides a good representation of the costs facing a firm recruiting manpower. Annex IV contains the relevant series for the various countries. The share is largest for Germany and France. The countries in which it has grown most are the United States, Germany and Canada.

36. The final specification of the hours-worked equation is thus(1):

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(1) The Table 4 results do not cover all the variables of (17), which is the most general equation possible.

$$(17)\quad H = H_{tend} + [a^{+}(TUC-\overline{TUC})^{+} + a^{-}(TUC-\overline{TUC})^{-} + b^{+}(\dot{X}-\bar{\dot{X}})^{+} + b^{-}(\dot{X}-\bar{\dot{X}})^{-}] \,.\, [c + d\, PAR]$$

where H = hours worked

H_{tend} = trend hours worked

TUC = capacity utilisation rate

$(TUC-\overline{TUC})^{+}$ = $TUC-\overline{TUC}$ when $TUC-\overline{TUC} > 0$ ($TUC-TUC^{-}$ when < 0)

X = output

PAR = non-wage share of total labour cost

37. In equation (17), trend hours worked are represented by a second degree polynomial in time, reproducing the way in which they stabilized in most countries during the late 1970s, following a period of rapid decrease. The results obtained for the various countries are grouped in Table 4. The non-wage component of the labour cost, which represents the effect of the employment adjustment cost, is significant for Germany and Japan but very weak in the United States(1). An asymmetry appears for the United States, Germany, Japan and Canada where, except for Japan, hours worked are more flexible upwards than downwards; this, according to the model, shows that overtime pay is not very high in comparison with the cost of short-time working. In the United States, the temporary lay-off system certainly reduces the downward flexibility of average hours worked. In Japan hours worked are virtually insensitive upward, but are very sensitive downward; the explanation might be that, since longer hours are worked in Japan than elsewhere, a constraint is being encountered in the form of the maximum possible work time.

38. Investment may be affected in two ways by a decline in trend hours worked. The first effect is that existing output capacity is reduced, which may encourage firms to invest more, so as to restore it; the other effect is that the capital/output ratio may rise, since the same quantity of output will require more equipment, individual equipments being less intensively used. An attempt was made to estimate investment equations introducing both effects of a decline in trend hours, but no significant results were obtained; in no country did investment seem to depend on trend hours worked.

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(1) Germany is a country in which this component is both very large, and has been growing considerably. By contrast, in Japan it is small and has tended to decline. So the results are not systematically linked to any particular evolution of these costs.

Table 4

Estimation of the Hours worked Equation

(Annual data 1964-1978)	Con-stant	T (Time)	T^2	I	II	III	IV	V	VI	R^2	DW	SEE (%)
United States	1876 (162.0)	-6.07 (3.3)	-0.21 (3.0)	436.7 (6.4)	253.7 (6.6)					0.998	3.15	0.18
Germany	1918 (40.8)	-12.14 (1.8)	0 (0)	266.3 (1.6)	244.2 (3.7)			286.1 (1.3)	124.8 (0.6)	0.998	2.67	0.23
France	2041 (69.5)	-17.67 (4.1)	0.10 (0.7)	827.0 (4.6)	133.1 (1.1)	829.4 (4.6)	133.5 (1.1)	169.4 (3.0)	185.6 (2.6)	0.999	2.80	0.18
United Kingdom	2226 (156.4)	-0.76 (0.3)	-0.80 (7.0)	310.7 (1.0)	479.0 (2.6)					0.997	2.26	0.33
Japan	2069 (84.0)	-17.70 (4.2)	0.19 (1.1)	1068.2 (2.9)	828.8 (3.1)					0.984	1.89	0.55
Canada	2539 (43.2)	-34.87 (4.5)	0.75 (3.0)	133.9 (1.7)	1405.0 (7.3)			227.5 (2.9)	296.6 (1.1)	0.999	3.24	0.11
	2137 (28.1)	-32.43 (2.8)	0.76 (1.8)			826.2 (2.0)	186.0 (0.9)			0.962	2.02	0.56

I : $(TUC-\overline{TUC})^+$

II : $(TUC-\overline{TUC})^-$

III : $(\dot{X}-\overline{X})^+$

IV : $(\dot{X}-\overline{X})^-$

V : $(TUC-\overline{TUC})^+ \, . \, PAR$

VI : $(TUC-\overline{TUC})^- \, . \, PAR$

39. The second possible impact of movements in work time is on employment. A fall in trend hours worked may have three consequences:

(i) output capacity may decline, because investment is not responding, as mentioned above;

(ii) labour productivity may increase as a result of the shorter working day, which would limit the loss of capacity;

(iii) firms may reorganize production through, for example, greater use of shift working. Employment then increases as a result of decline in trend hours, which again makes it possible to produce more with existing equipment.

40. The latter point was tested by taking the employment equations (11B) or (12B) from Part II and adding an effect of the non-cyclical component of hours worked (H_{tend}) estimated with (17). The equation takes the form:

$$(18)\quad \log N = \lambda \log N_{-1} + (1-\lambda)[\log N^* + b \log TUC + c \log H_{tend} + d]$$

where N = number of persons employed

N* = optimum number of employees with full capacity working (see Part II)

TUC = capacity utilisation rate

The results are given in Table 5 and show that in all countries, the decline in trend hours worked significantly increases employment: for the United States, France, the United Kingdom and Canada, the coefficient is near to 1, showing that in those countries there is no productivity gain and that reorganising production fully offsets the initial loss of capacity. In Germany and Japan the coefficient is significantly smaller than 1, suggesting either that the reduction in hours worked permitted productivity gains, which is plausible for Japan in view of the original level of hours worked, or that some loss of capacity persists.

41. Cyclical movements in hours worked should also have an influence on employment. When output rises in the short term, two factors reduce the effect on the number of employees:

(i) more hours are worked;

(ii) short-term labour productivity goes up (faster work pace, attempts to improve efficiency, etc.).

Table 5

Equation of numbers employed with trend hours worked

(Annual data 1964-1978)

	λ	b (TUC)	c (H_{tend})	d (Constant)	R^2	DW	SEE (%)
United States	0.42 (2.2)	0.70 (3.3)	-1.03 (4.4)	7.69 (4.3)	0.994	2.30	0.75
Germany	0.53 (8.7)	1*	-0.71 (3.8)	5.39 (3.9)	0.943	0.76	0.93
France	0.46 (4.4)	0.80 (5.7)	-0.97 (19.7)	7.30 (19.4)	0.947	1.43	0.42
United Kingdom	0.72 (4.7)	1.33 (1.4)	-1.06 (2.0)	8.04 (1.0)	0.947	2.37	1.01
Japan	0.25 (1.0)	0.22 (2.0)	-0.73 (4.0)	5.57 (4.0)	0.993	2.36	0.74
Canada	0.71 (6.9)	1*	-0.89 (2.0)	6.78 (2.0)	0.991	1.38	1.33

* set a priori, according to (11B).

As has been shown above, the cyclical response of hours worked can be asymmetrical (upwards and downwards); the same might therefore be true of the cyclical response of numbers employed. To test this idea the employment equation (18) was changed by introducing the ratio of the cyclical components of hours worked to the non-cyclical components (HC/H_{tend}), distinguishing their positive and negative values. In all countries, changes in the cyclical component of hours worked attenuated changes in employment. The coefficient was always greater than 1. This means that short-term productivity changes accompany changes in hours worked so as to stabilize employment. In many instances, the coefficients are about two or three, meaning that such changes in productivity are more important than changes in hours worked in explaining manpower rigidity. A very significant asymmetry appears for the United Kingdom and Japan, surprisingly for the former as no asymmetry had been detected in the work-time response(1). The result for Japan is quite consistent with the result obtained above, to the effect that hours worked are inflexible upwards. Only the temporary increase in productivity plays a role.

42. Changes in hours worked can influence one final aspect of behaviour, namely the way in which hourly wage rates are set. If workers wish to maintain their weekly or monthly earnings when trend hours worked decline, they must obtain an offsetting increase in the hourly wage rate, which will therefore be influenced negatively by trend hours. Moreover, cyclical lengthening of hours worked, meaning that more overtime is being worked, should entail an increase in the average hourly wage. The two hours-worked components were introduced in the hourly wage rate equation for the six countries studied. The only significant result obtained was for Japan, where cyclical lengthening of hours worked equal to 1 per cent of the trend level raised the average hourly wage rate, according to the estimate, by 0.3 per cent. No other positive result was obtained, either because of statistical problems or perhaps because workers had not been obtaining compensation for declines in hours worked.

V. Profits

43. The equations up to now have been derived purely from the original putty-clay model. In particular, they take no account of profits, whose significant influence on investment has been identified in many studies [Helliwell-Glorieux (1970), Le Marois (1979), Gardner-Sheldon (1975), Sarantis (1979), Coen (1971), Metric (1981)].

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(1) This may reflect changes in the composition of output, each sector having a different hours worked/employment trade-off.

44. One theoretical justification for the inclusion of profits in the determination of investment is the possibility that firms are under a solvency constraint [Malinvaud (1981), Artus-Sterdyniak (1980), Courbis (1973)]. Their indebtedness at the end of the period must be such that during the following period they can repay maturing loans and pay financial costs out of their own profits, allowing for a possible increase in indebtedness due to inflation. This gives:

$$(19) \quad END\ (\eta + i) = \pi^a\ p^a\ Q^a + \dot{p}^a\ END$$

where END = indebtedness

η = proportion of borrowings to be paid off

i = interest rate

π^a = expected profitability rate for subsequent period (profit/output ratio)

p^a = expected output price

Q^a = expected output

since END = $END_{-1} + p_I I - \pi p\ Q$

with p_I = investment cost

I = investment

This gives:

$$(20) \quad I \leqslant \left[\pi^a (1+p^a) Q^a / (\eta + i - \dot{p}^a)\right] .\ p/p_I - END_{-1}/p_I + \pi Q\ p/p_I.$$

Possible investment is an increasing function of the observed and expected profit rate, of the output price relative to the investment price, of output and of expected output, and a decreasing function of the real interest rate and the real value of inherited indebtedness. If some firms find this a constraint, then the variables having a bearing on it, particularly profits, can reasonably be added into the basic investment equation such as (B1) or (C1) in Annex 1. Another possible way of adding the profitability rate to the investment equation would be to start with a model in which financial markets are imperfect and in which a firm increasing its indebtedness does so by borrowing at increasingly high interest rates [see Milleron (1970)].

45. Another theoretical reason for introducing profits, which then influence the speed with which investment adjusts to its desired level, is to take account of adjustment or uncertainty costs. Introducing capital adjustment costs makes

the adjustment speed depend on interest rates and the real wage rate [Schramm (1970), Mortensen (1973), Eisner-Strotz (1963), Gould (1968), Lucas (1976), Craine (1975), Rothschild (1971)]. However, this specification adapts poorly to the putty-clay model, in which the adjustment cost should fall on new investment. Another possibility is the introduction of uncertainty [Nickell (1977), Hartman (1976)]. An elementary model could be constructed on the basis that firms are uncertain about the level of demand and must anticipate a high marginal rate to accept the risk of over-capacity, employment being inflexible downwards, and to cover the financial outlay on the investment. In this case, the speed at which investment adjusts is a positive function of the profit rate, and a negative function of the nominal and real interest rate.

46. When profits were added directly to equation (B1) or (C1) deriving from the three-factor putty-clay production function following the first (solvency) model, the only significant result obtained was for Canada, where profits(1) appeared to play a considerable role in explaining investment in recent years(2). For no other country were the profit variables significant. They could be added significantly to a simple accelerator, but as soon as the relative prices for the three factors were involved the significance disappeared. This rather puzzling phenomenon was observed several times. It might be due to coincidence of oil price rises, which lead in most countries to a substitution of labour for the capital energy bundle out-weighing(3) that of capital for energy, with a fall in firms' profits. For three countries, however, the uncertainty model could be applied significantly, with the effect of varying the investment lag. The countries concerned were the United States, France and Germany. When X represents the deviation of the profits/output ratio from the mean, the following values were obtained for the accelerator coefficients:

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(1) Defined as the capital goods purchasing power of firms' gross savings.

(2) The coefficients obtained are:

r (technical progress): 0.033
(3.9)
capital/energy elasticity of substitution: 0.66
(1.9)
coefficient of profits: 0.44
(6.1)
$R^2 = 0.98$ DW = 1.25 SEE = 4.7

(3) This was found to occur in all countries except the United Kingdom and Germany.

	United States		Germany		France	
a_0	0.28 +	0.10 X	0.30 +	0.28 X	0.42 +	1.38 X
	(5.4)	(0.8)	(11.5)	(12.4)	(8.6)	(1.9)
a_1	0.30 +	0.38 X	0.30 +	0.14 X	0.30 -	1.38 X
		(2.0)		(8.7)	(6.4)	(1.9)
a_2	0.26 +	0.13 X	0.25 -	0.07 X		0.28
	(9.2)	(0.9)	(19.2)	(2.3)		
a_3	0.16 -	0.51 X	0.15 -	0.35 X		
	(5.4)	(2.2)	(11.5)	(11.3)		

In those three countries a rise in the profit rate did indeed shorten the average investment lag following changes in output, though it remained neutral over the medium term.

VI. Other effects of firms' profitability

47. How does the model behave if the price of variable inputs rises in relation to the output price? Scrapping is intensified and more old equipment is replaced by new equipment. In the short term, output capacity is reduced, but when enough time has elapsed for the investment to be implemented, we return to a Keynesian regime in which demand governs capacity since the fall in profits has no influence in the model on the investment level in the medium term, except in Canada. If there are medium-term supply effects in this model, they are being transmitted indireclty via the impact of price rises on the level of demand, thereby affecting installed capacity. If this indirect effect is ignored in the present state of the supply block, demand is exogenous and never rationed in the medium term by insufficient profitable capacity (except in Canada, where insufficient profits may be reducing investment).

48. There is no question here of estimating a genuine disequilibrium model, which might reveal some demand rationing pattern. Demand, however, can be separated into changes in stocks and demand excluding stocks, the changes in stocks component being controllable by the firm itself while the other component is beyond the firm's control and exogenous. If the variable cost share in the value of output rises, firms may begin not only to scrap more equipment, but also to stop using a certain amount of installed capacity, perhaps thereby building up fewer stocks or satisfying demand from stocks and not from unprofitable marginal output(1). From this, a capacity utilisation rate equation can be estimated which is an implicit equation of changes in stocks, linking the rate to

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(1) This concept of a stock/output breakdown and therefore of choice as to capacity utilisation rates, is discussed in Helliwell-McRae (1980).

demand excluding stocks, to the variable cost share in output and to the desired change in the stock level (i.e. in the absence of a profitability condition and a buffer-stock effect) as follows:

$$(21) \quad X/XPOT = a + b\ DHS/XPOT + c\ (NS^* - NS_{-1})/XPOT + d\ (pvV/pX)$$

where X = production
XPOT = potential output
DHS = demand excluding stocks
NS* = desired stock level
NS = actual stock level
pvV = variable costs
pX = output value

Assuming: $NS^* = nDHS$

gives the estimated equation:

$$(22) \quad X/XPOT = a + (b + cn)\ DHS/XPOT - c\ NS_{-1}/XPOT + d\ (pvV/pX)$$

49. In an open economy stock and import changes could be treated symmetrically, in which case DHS represents total domestic demand excluding stocks; if, however, firms know what proportion of demand excluding stocks is satisfied by imports, and decide how much to produce by reference to the remainder, in (22), DHS then becomes total demand excluding stocks and excluding imports(1) (Table 6). The potential output variable used is that resulting from the estimation of the model for various countries, with endogenous scrapping rates.

50. For every country but France(2), total demand excluding imports led to better results, both econometrically and from the standpoint of the plausibility of coefficient. This would suggest that firms in all those countries have a fairly accurate idea of how the domestic market is shared between domestic output and imports before determining their

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(1) The imports and stocks used cover the full range of products. It might have been preferable to exclude raw materials not produced domestically.

(2) In France, therefore, imports and changes in stocks are probably determined fairly symmetrically. We also observed that the import content of stocks was high, which explains why there is some degree of parallel movement in the two series.

Table 6

Estimation of the utilisation rate equations

(Annual data 1964-1979)

		DHS/XPOT						
	Constant	Including imports	Excluding imports	$\frac{NS_{-1}}{XPOT}$	$\frac{pvV}{pX}$	R^2	DW	SEE
United States	0.22 (3.1)	0.97 (38.6)		0.51 (1.7)	-0.59 (15.5)	0.98	1.11	0.006
	0.07 (1.0)		1.10 (43.2)	-0.35 (1.3)	-0.10 (2.7)	0.98	1.22	0.005
Germany	0.92 (10.4)	0.73 (7.7)		-0.39 (0.9)	-1.20 (12.4)	0.90	0.78	0.012
	-0.16 (1.2)		1.27 (9.9)	-0.39 (1.1)	0.03 (0.2)	0.94	1.43	0.010
France	0.84 (9.4)	0.41 (5.1)		-0.10 (2.8)	-0.50 (5.8)	0.91	0.62	0.007
	0.55 (2.0)		0.64 (2.7)	-0.05 (1.4)	-0.27 (1.8)	0.87	0.60	0.009
United Kingdom	-0.11 (0.5)	0.66 (5.4)		0.78 (5.5)	-0.16 (1.9)	0.81	0.61	0.014
	-0.15 (1.1)		1.36 (8.5)	-0.30 (1.9)	-0.03 (0.5)	0.89	1.54	0.010
Japan	0.30 (1.7)	0.77 (9.6)		-0.33 (0.8)	-0.23 (5.3)	0.95	1.49	0.005
	0.52 (4.0)		0.80 (11.3)	-1.07 (3.4)	-0.17 (4.5)	0.96	1.80	0.005
Canada	0.62 (9.1)	0.69 (42.6)		0.54 (6.0)	-1.18 (10.6)	0.99	1.99	0.006
	0.25 (2.2)		1.04 (25.5)	-0.13 (0.8)	-0.47 (2.6)	0.99	1.14	0.009

production plans. The share of variable costs in output was significant in the equations for the United States, France, Japan and Canada. For Germany, it was also significant in the equation with total demand including imports. Generally speaking, output profitability does, therefore, seem to influence capacity utilisation rates, thereby influencing stocks in the formalization adopted. It would doubtless be hasty to conclude that any real self-rationing mechanism is at work here when installed capacity becomes not very profitable. This is because greater or lesser availability of internal funds may also have a bearing on desired stock levels, firms being reluctant to borrow too heavily to finance stocks when profits are low. One last point about the estimations is that stocks adjust to desired levels instantaneously in Japan, moderately fast in the United States, Germany and the United Kingdom, and very slowly in France and Canada(1).

51. The model can also be refined as regards the effect of output profitability on employment, making a second test(2) for demand exogeneity, i.e. the assumption that firms do not maximize profits over the medium term. If they did maximize profits, the employment allocated to each new vintage of equipment would depend only on the costs of each input in relation to the output price. Total employment would depend on the entire historical trend of real factor costs. This is very complicated to specify accurately. We therefore estimated a rather "ad hoc" equation taking the specification (18) drawn from cost minimization and adding first the logarithm for the real wage cost and second the variable cost share in output, as follows:

$$(23) \quad \log N = \eta \log N_{-1} + (1-\eta)\left[\begin{array}{l}\log N^* + b \log TUC \\ \quad + c \log (H_{tend})\end{array}\right] + \left(\begin{array}{l}\log \frac{w}{p} \\ \frac{pvV}{pX}\end{array}\right.$$

where N^* = optimal employment calculated above

TUC = capacity utilisation rate

H_{tend} = trend hours worked.

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(1) This can be seen from the coefficient of $NS_{-1}/XPOT$, which is minus c, being the proportion of the stock shortfall made up during the period. The coefficients obtained for Canada by Helliwell-McRae ($c = 0.17$, $d = -0.48$) are close to those found here.

(2) The first, briefly described above, starts from output price formation.

If some firms maximize their profits, employment should show sensitivity to the real wage rate appearing in (23). Moreover, adding the share of variable costs in output allows for the possibility that changes in the variable cost share will have a greater impact on employment than on replacement investment. A firm may give up using equipment without replacing it, but this is taken into account by the fall in the capacity utilisation rate TUC; on the other hand, that kind of equipment is doubtless the most intensive user of variable factors, so that taking it out of work has a more than proportional effect on employment and energy consumed, which justifies the addition of pvV/pX. Clearly, (23) is far from being a rigorous formulation and can only be used to suggest some of the consequences of movements in profitability.

52. It can be observed (Table 7) that the real wage rate is never significant. The presence of the relative cost of labour and of other production factors in desired employment is sufficient to account for fluctuations in labour productivity, so there is no need to bring in the real wage resulting from profit maximization. The variable cost share in output was significant in two countries: Germany and Canada in which, as mentioned above, the scrapping rate rises the most after an increase in the variable cost share. The sensitivity of scrapping behaviour accounts for the result obtained here, because the fact that many older, labour-intensive machines may be scrapped should normally show up in total employment, as was indeed the case.

VII. Final model and simulations

53. The final model is the outcome of combining all the changes and improvements progressively made to the initial model of Part II: flexibility of scrapping, effects of hours worked and profits. It was also re-estimated for all the countries using half-yearly data. The basic parameters of the final equations are given in Table 8.

54. Estimating the model with half-yearly data made it possible to calculate potential output and capacity utilization rate series, according to the methodology defined in Part II, for the six countries. These series are reproduced in graph form in Annex 5 and are compared with those constructed by the International Monetary Fund using the method developed by Artus (1977)(1). Because of the lags introduced in the estimates and because certain semi-annual series were not available for the early years for some countries (Japan, France), it was only possible to calculate the capacity utilization rate over a fairly short period (starting 1968 or 1971). Owing to differences in the methods of constructing the two capacity utilization rate series,

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(1) Based on estimation of putty-putty production functions.

Table 7

Estimation of the Employment Equation with the Additional Effect of Real Wages or of Cost Share

		b	c	log w/p	$\frac{pvV}{pX}$	R^2	DW	SEE (%)
United States	0.78 (37.0)	1*	-0.95 (2.0)		-0.04 (0.9)	0.99	2.35	0.73
	0.78 (38.1)	1*	-0.60 (1.8)	0.01 (0.9)		0.99	2.16	0.75
Germany	0.40 (6.9)	0.69 (7.6)	-0.98 (8.2)		-0.15 (2.2)	0.97	1.83	0.78
	0.34 (6.0)	0.70 (7.6)	-0.89 (5.4)	-0.01 (0.6)		0.96	1.49	0.92
France	0.64 (24.7)	1*	-0.99 (3.9)		0.02 (0.6)	0.95	1.44	0.46
	0.68 (29.4)	1*	-0.99 (0.6)	0.003 (0.1)		0.95	1.44	0.47
United Kingdom	0.82 (17.2)	1*	-1.02 (1.8)		0.01 (0.2)	0.95	2.12	1.04
	0.80 (16.5)	1*	-1.05 (2.0)	0.006 (0.5)		0.95	2.15	1.03
Japan	0.74 (20.7)	1*	-1.01 (3.2)		0.08 (1.8)	0.99	1.67	0.52
	0.78 (23.4)	1*	-0.87 (3.2)	0.015 (1.6)		0.99	1.60	0.53
Canada	0.82 (17.4)	1*	-0.90 (2.0)		-0.73 (2.6)	0.99	1.56	1.33
	0.81 (17.2)	1*	-0.85 (2.0)	-0.04 (0.9)		0.99	1.47	1.62

(*) A priori.

Table 8

Parameters of the final equations

	Trend of technical progress	Elasticity of substitution K/E	Elasticity of substitution KE/L	Direct effect of a 1% increase in profits on investment	Employment effect of a 1% fall in trend hours worked	Effect on scrapping rate of a 1% increase in the variable share costs
United States		0.25	1	0	+0.6 %	+0.2 pt/year
Germany	+2.2 %/year	0.63	0.80	0	+0.8 %	+0.3 pt/year
France	+3.1 %/ "	0.16	1	0	+1.0 %	0 in medium term +0.1 pt in short term/year
United Kingdom	+0.2 %/ "	0.20	0.15	0	0	0
Japan	+2.8 %/ "	0.07	0.80	0	+1.0 %	0 in medium term, +0.2 pt in short term/year
Canada	+6.0 %/ "	0.66	1	+0.4 %	+1.1 %	+0.04 pt/year

their average absolute levels are not comparable. However, the trend of their levels can be compared over the common period. There is no very marked divergence between the two for the United States and Germany. For France, the United Kingdom, Canada and especially Japan, the series calculated here show a much smaller decline than the IMF series. Such divergences are quite reasonable since the fact that the trend of relative prices is taken into account results in reproducing a rise in the capital/output ratio, hence a reduction in the capacity created for a given amount of investment. On the other hand, the fluctuations in the two series are very similar for the United Kingdom, the United States and Germany. For Canada, there is a divergence from 1965 to 1967, but thereafter the two series are very similar. The series obtained here for France and Japan show a much smaller decline at the time of the first oil shock than those calculated by the IMF. This is perhaps due to the endogenization of the scrapping rate which results in a reduction in capacity following the rise in the share of costs in output which took place in 1974 and 1975.

55. A comparison was also made between the utilization rate series obtained here and those deriving from national business survey sources, namely the CBI for the United Kingdom(1), INSEE for France, MITI for Japan, the Federal Reserve Board for the United States, IFO for Germany, and the Ministry of Supply and Services for Canada (Annex 5). These series are also very similar to those constructed by the putty-clay model so far as fluctuations are concerned. For Japan, however, the national series shows much wider fluctuations and in Canada there is the same divergence as with the IMF series over the period 1965-1966. Differences in the trend of the levels of the series are in every case smaller than with the IMF series.

56. Using the utilization rate series constructed from the putty-clay model, some simple equations of the "Okun"-type were estimated:

$$UNR = a + b\ GAP$$

where UNR is the unemployment rate.

The results tabulated below show that the fairly strong similarity between the utilization rate and unemployment rate series up to 1974 disappears after 1975. The differences between the observed and fitted unemployment rate series suggest that unemployment cannot be eliminated by a return to normal capacity utilization. The average divergence from 1971 to 1979 was 1.3 percentage points for the six countries taken

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(1) This survey in fact gives the percentage of firms working at full capacity, which is assimilated to a utilization rate.

Table 9

Estimation of the Okun equation

	Constant	Utilisation rate'	R^2	DW	SEE	Average error(1)(%) in				
						1971	1973	1975	1977	1979
United States	23.9 (5.9)	-20.2 (4.6)	0.24	0.04	1.27	-0.4	-0.2	+1.6	+1.9	+0.7
Germany	27.7 (5.5)	-27.8 (5.2)	0.49	0.15	1.04	-0.8	-0.9	+0.2	+1.8	+2.0
France	53.0 (10.9)	-53.2 (10.2)	0.73	0.26	0.74	0	0	-0.9	-0.1	+1.4
United Kingdom	36.6 (6.7)	-37.6 (6.0)	0.56	0.31	1.16	-0.6	-0.2	-1.6	+1.2	+1.1
Japan	91.2 (36.7)	-1.39 (1.0)	0.06	0.59	2.26	-0.5	+3.2	-3.4	+0.4	+1.7
Canada	13.34 (3.7)	-7.82 (2.0)	0.13	0.10	1.47	+0.6	+0.3	+0.8	+1.8	+0.8
Average						-0.3	+0.4	-0.5	+1.2	+1.3

(1) Observed unemployment rate - unemployment rate calculated from the estimates for the equation UNR = a + b GAP.

together. The difference was particularly big for Germany, France and Japan and small for the United States and Canada (Table 9).

57. The supply block estimates can be used to simulate the effects of changes in factor prices. The trend of the labour-capital relative price and the energy-capital relative price is shown in Table 10. Short-term fluctuations in these relative prices are due mainly to those in the costs of capital use and, more specifically, in real interest rates. In the medium term, the cost of capital use moves with the price of investment, and the ratios calculated come close to the real wage and the real energy price(1). The labour-capital relative price moves quite differently depending on the country. It rises very steeply in France and particularly Japan, fairly steeply in Germany and the United Kingdom, but very little in the United States and Canada. In the last two countries the real wage cost (expressed as a ratio to the value-added price) grew by only 16.2 and 15.1 per cent respectively between 1968 and 1979, which explains the small rise in the relative price. The energy-capital relative price shows a fairly similar trend in all countries except Germany, where it remains virtually flat. The price of intermediate energy consumption rose by 81 per cent in Germany between 1968 and 1976, the corresponding figure for the United States, for example, being 400.5 per cent. This is doubtless partly due to the large share of domestically-produced coal in Germany's energy consumption and the appreciation of the Deutschmark against the dollar.

58. Two simulations were made on the supply blocks(2): one in which it was assumed that the nominal energy price remained constant between 1971 and 1979 at its beginning-1971 level and another in which it was assumed that the real wage-cost per hour (i.e. deflated by the value-added price) rose on average during this period at the same pace as labour productivity, which is to assume that on average the share of wage cost in value added remained constant. The results obtained are grouped in Table 11, which gives deviations from the historical trend caused by the stability of the energy price and by the change in the wage cost.

59. The consequences of energy price stability depend on the characteristics of the production functions and on the scale of the movement in the relative energy price. In

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(1) i.e. with respect to the value-added price.

(2) For these simulations we added to the factor demand equations the identity from which the gross domestic product is calculated, so that investment movements would have an influence on value added, with a feedback to investment itself. On the other hand, the other components of demand or profits will not be affected by movements in employment or in the cost share in output.

Table 10

Trend of relative factor prices

(1968 = 100)

	United States		Germany		France		United Kingdom		Japan		Canada	
	I	II	I	II	I	II	I	II	I	II	I	II
1968	100	100	100	100	100	100	100	100	100	100	100	100
1970	96.7	92.3	111.0	88.8	114.5	95.5	118.3	100	132.1	95.1	100.2	97.2
1973	106.6	105.6	143.1	99.3	164.5	118.7	126.3	88.7	218.2	107.7	115.3	106.1
1975	109.8	170.3	164.5	131.3	194.5	190.1	190.8	177.0	335.0	286.3	144.8	147.7
1977	117.0	217.3	179.7	134.1	185.5	175.2	262.6	274.6	280.9	214.5	141.1	169.3
1979	112.8	223.2	170.2	138.0	229.3	197.7	148.1	140.0	272.2	190.0	118.3	162.2

Note: I. Labour-capital relative price (ratio of labour cost to cost of capital use).

II. Energy-capital relative price of energy and capital (ratio of energy price to cost of capital use).

Table 11

Simulations - divergences (%)

	United States								Germany(a)								France							
	Constant energy price				Constant breakdown of value added				Constant energy price				Constant breakdown of value added				Constant energy price				Constant breakdown of value added			
	I	E	EN	PP	I	E	EN	PP	I	E	EN	PP	I	E	EN	PP	I	E	EN	PP	I	E	EN	PP
1972	+0.2	0	+0.1	0	0	+0.2	+0.1	+0.2	-0.2	-0.1	+0.2	0	+0.2	0	0	0	+2.6	0	+1.8	0	-2.1	-0.1	-1.7	+0.2
1974	+1.9	-0.1	+1.9	-0.1	-3.8	0	-0.7	+0.5	-3.3	-0.8	+3.3	+0.3	-1.7	+0.1	-0.3	0	+12.6	-0.3	+9.1	+0.3	-4.3	0	[illegible]	[illegible]
1975	+5.8	-0.1	+5.1	-0.2	-2.9	0	-0.8	+0.1	-4.3	-1.4	+6.3	-0.5	-1.0	+0.3	-0.4	+0.2	+24.5	-0.8	+18.5	+0.5	-7.0	+0.1	-4.4	+0.3
1977	+10.7	-0.6	+13.8	+0.5	-2.2	+0.1	-0.5	-0.3	-4.6	-2.7	+13.2	+1.0	-0.7	+0.5	-0.7	+0.4	+34.1	-3.4	+33.4	+2.6	-6.2	+0.7	-5.1	-0.7
1979	+13.2	-1.5	+25.3	+1.7	-1.9	+0.3	-0.6	-0.1	-5.0	-4.1	+21.1	+1.6	-0.4	+0.5	-0.8	+0.5	+40.4	-6.9	+45.8	+4.0	-3.5	+1.8	-3.8	[illegible]

	United Kingdom								Japan								Canada							
	Constant energy price				Constant breakdown of value added				Constant energy price				Constant breakdown of value added				Constant energy price				Constant breakdown of value added			
	I	E	EN	PP	I	E	EN	PP	I	E	EN	PP	I	E	EN	PP	I	E	EN	PP	I	E	EN	PP
1972	+0.4	+0.1	+0.3	+0.1	-0.3	0	0	0	-0.1	0	0	0	-4.5	-0.1	-3.2	+1.6	0	0	0	0	+0.1	+0.2	0	-0.1
1974	-0.8	0	+0.3	+0.1	-2.4	0	-0.3	0	+6.7	+0.1	+1.2	+1.0	-12.0	-0.1	-8.4	-1.0	0	-0.1	+0.9	0	-0.2	+0.2	+0.1	+0.2
1975	-1.2	0	+0.7	+0.1	-2.0	0	-0.6	0	+14.5	+0.1	+5.1	+1.4	-13.1	+0.2	-9.3	-0.2	-0.1	-0.4	+2.5	+0.1	+0.1	+0.6	+0.2	+0.6
1977	-0.8	+0.1	+2.5	+0.1	0	0	-0.7	0	+21.4	-1.4	+14.1	+1.6	-10.4	+1.5	-6.5	-1.3	+0.1	-1.1	+8.3	+0.1	+0.2	+1.1	+0.4	[illegible]
1979	-0.9	+0.2	+5.6	+0.2	+0.2	0	-0.9	+0.2	+23.9	-4.0	+20.3	+3.1	-5.1	+3.3	-2.1	-2.4	+0.4	-2.4	+18.5	+0.2	-1.0	+1.0	+0.3	+2.4

I = Investment EN = energy E = employment PP = potential output

(a) Provisional figures.

Germany and the United Kingdom, the elasticities of substitution are such that, on balance, investment and energy are substitutable; stability of the energy price at a low level therefore reduces investment in these two countries. In all the others, investment is stimulated, only a little in Canada because of the importance of profits in the explanation of investment(1) and the fairly small movement in relative prices, but strongly in France and Japan where capital-labour substitution easily outweighs capital-energy substitution. In all countries except the United Kingdom, rises in the energy price reduced gains in labour productivity. In the United Kingdom, the elasticity of substitution between labour and the capital-energy bundle is so low (0.15) that no significant movement shows up in employment, which is however affected by movements in output via the capacity utilization rate. Average annual productivity losses since 1974 due to the rise in energy prices range from 0.4 per cent in the United States and 0.6 per cent in Canada to 1 per cent in Japan and Germany and 1.6 per cent in France. Movements in potential output in all countries are linked, first, to investment movements which feed back to the desired productive capacity and, second, to any fall in the scrapping rate due to energy price stability, particularly apparent in the case of the United States, Germany and Japan (see Table 8). The loss in productive capacity caused by the oil crises thus seems negligible for the United Kingdom and Canada, fairly large for the United States and Germany and very large for Japan and France, with a figure of almost 1 per cent per year since 1974.

60. To ensure that on average the value-added breakdown remained constant, we had to adjust wage cost in the proportions shown in Table 12. In all countries wage cost had to be reduced considerably between 1973 and 1976, and by a smaller proportion in subsequent years. However, the adjustments varied a great deal across countries: they were small for the United States, Germany and Canada, very large for France, the United Kingdom and especially Japan where, in addition, the wage cost share in value increased throughout the period to 1979, which meant that wage cost had to be adjusted continuously and in large proportions. The effects of stability of the value-added breakdown therefore also varied considerably across countries. The fall in the cost of labour, which was general, caused a decline in investment and energy consumption, an increase in employment, but also

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(1) Because profits appear in the investment equation, in Canada a change in relative factor costs directly affects productive capacity. The reason is that such a change only has a limited effect on investment, part of investment arising from profits, but has its full impact on the optimal capital/output ratio since enterprises choose the best technology. There is thus an effect on productive capacity.

Table 12

Adjustments made to wage cost (%)

	1971	1972	1973	1974	1975	1976	1977	1978	1979
United States	+0.1	-1.4	-2.5	-1.0	0	-0.9	-1.4	-1.5	-1.6
Germany	+0.4	+0.5	-1.2	-3.4	-2.8	-2.1	-2.0	-1.4	+1.5
France	-2.3	-3.1	-3.8	-7.1	-11.6	-12.8	-11.6	-9.9	-9.6
United Kingdom	+1.5	-1.3	-8.2	-12.1	-12.6	-10.5	-4.5	-5.3	-4.8
Japan	-1.4	-7.5	-13.1	-14.5	-18.7	-19.0	-20.2	-18.3	-17.2
Canada	+0.2	-0.7	-1.0	-1.8	-4.6	-6.7	-7.1	-5.6	-2.0

a decrease in scrapping which reinforced the decline in investment, although it meant that initially greater productive capacity could be maintained, which was particularly noticeable in Germany and Canada. The low elasticity of substitution between labour and the capital-energy bundle in the United Kingdom rendered the effects on employment negligible there. In all the other countries the shift in the breakdown of value added would have resulted, according to the model, in a decline in employment in 1979, ranging from 0.3 per cent in the United States to 3.3 per cent in Japan. The positive consequences of this shift for investment were particularly marked for Japan and France; for Canada they were very small because of the role played by profits in the determination of investment.

VIII. Summary and conclusions

61. From this set of estimates we were able to produce a complete and consistent supply model for the six major countries. The salient points are these:

- in spite of the similarity of the models used, there were wide differences across countries in the responses to relative price movements: there was virtually no labour substitutability in the United Kingdom and a high elasticity of substitution between labour and the capital-energy bundle everywhere else; the elasticities of substitution between capital and energy ranged from 0.1 to 0.6; scrapping behaviour was both very active (Japan, Germany) and very sluggish (France, the United Kingdom);

- certain common features did emerge, however: first, employment seemed to be positively affected in all the countries by a fall in trend hours worked; second, after allowance was made for the impact of relative price movements, the influence of profits proved very small, except in Canada.

62. Many improvements could be made to these supply models: expectations could be better represented; stocks and hours worked could be introduced in the initial model, instead of at a second stage; further research could be done on scrapping and an attempt could be made to calculate a scrapping rate by factor of production; taxation and the expected lifetime of the capital stock could be better represented in the calculation of the series for the cost of capital use. It might be useful, at a subsequent stage, to introduce a distinction between the two forms of energy (oil and non-oil) and certain substitutions between energy categories having possible major macro-economic consequences.

ANNEX 1

Minimisation of expected cost subject to the production function constraint

1. When a firm installs new equipment it minimises the expected total cost of output, discounted at the rate i, subject to the constraint of demand satisfaction:

$$\text{Min } F_t p_t K_t + \sum_{\theta=1}^{T} \frac{W_{t+\theta} L_{t+\theta} + e_{t+\theta} E_{t+\theta}}{(1+i)^{\theta}} (1-v)$$

$F_t p_t K_t$ represents the cost of buying equipment (K) at the price p, and allowing for possible tax emissions whose effect on the purchase price is represented by F.

T is the expected lifetime of the equipment.

$W_{t+\theta} L_{t+\theta}$ is the wage cost in period t+θ, with $W_{t+\theta}$ the wage cost (including social insurance); $e_{t+\theta} E_{t+\theta}$ is the cost of energy consumption, at the price e. These costs are deductible for profits tax purposes, the apparent tax rate being v, and allowance must therefore be made for such a deduction by multiplying costs by 1-v; v also represents (because it is an apparent rate calculated *ex post*) the effect of fiscal depreciation on taxes.

2. In order to simplify the exercise, it is assumed that there is no disembodied technical progress, so that L and E remain as they were on the date of installation. The expected cost will therefore be written:

$$\text{(A)} \quad C_t = F_t p_t K_t + W_t L_t \sum_{\theta=1}^{T} \frac{W_{t+\theta}/W_t}{(1+i)^{\theta}} (1-v) + e_t E_t \; .$$

$$\sum_{\theta=1}^{T} \frac{e_{t+\theta}/e_t}{(1+i)^{\theta}} (1-v)$$

The additional simplifying assumption will be made that firms expect equipment to have a fixed lifetime(1). Minimisation of C_t subject to the constraint of either of the two production functions given earlier leads to the expressions for the optimal investment, employment and energy demand assigned to the new vintage.

3. From (A) we can write the expected cost:

$$C_t = F_t p_t k_t + w_t L_t \sum_{\theta=1}^{T} \frac{(1+\dot{w})^{\theta}}{(1+i)^{\theta}} (1-v) + e_t E_t \sum_{\theta=1}^{T} \frac{(1+\dot{e})^{\theta}}{(1+i)^{\theta}} (1-v)$$

where K = investment, p its price, F the tax effects on investment

L = labour, w its cost, $\dot{w}$ the expected rate of growth of its cost

E = energy, e its price, $\dot{e}$ the expected rate of growth of its price

i = the nominal interest rate

v = average apparent rate of direct taxation

T = expected lifetime of the equipment,

or:

$$C_t = F_t p_t K_t + w_t L_t \frac{1-\left(\frac{1}{1+i-\dot{w}}\right)^{T}}{i-\dot{w}} (1-v) + e_t E_t \frac{1-\left(\frac{1}{1+i-\dot{e}}\right)^{T}}{i-\dot{e}} (1-v)$$

Minimising C_t is equivalent to minimising C'_t:

$$C'_t = \frac{F_t}{1-v}\, p_t \frac{i-\dot{w}}{1-\left(\frac{1}{1+i-\dot{w}}\right)^{T}} K_t + w_t L_t + e_t E_t \frac{i-\dot{w}}{i-\dot{e}} \frac{1-\left(\frac{1}{1+i-\dot{e}}\right)^{T}}{1-\left(\frac{1}{1+i-\dot{w}}\right)^{T}}$$

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(1) In the article referred to, Ando *et al*. perform the optimisation also on the expected lifetime. In the case of a Cobb-Douglas production function, they show that it is only dependent on the parameters of the function and the expected rates of growth of p, W and e. But this would lead to excessively complicated expressions, particularly with more complex production functions and it will therefore be assumed that T is fixed *ex ante*.

where the cost of capital use u is defined as

$$u = \frac{F_t}{1-v} p_t \frac{i-\dot{w}}{1-(\frac{1}{1+i-\dot{w}})^T}$$

We shall therefore also in principle have to define a cost of energy use u_e equal to:

$$u_{e_t} = e_t \frac{i-\dot{w}}{i-\dot{e}} \frac{1-(\frac{1}{1+i-\dot{e}})^T}{1-(\frac{1}{1+i-\dot{w}})^T}$$

To simplify the expression, we used a single expression for the expected real rate of interest. Thus we set $i-\dot{e} = i-\dot{w}$, from which we get $u_e = e$. This is of course an arguable procedure, but it is necessary to think in terms of this expectation relating to the whole lifetime of the equipment, which perhaps makes the simplification less unacceptable. The effects of the relative energy price would always be transmitted in the model via the relationship e/u(1). Firms' behaviour is thus as follows:

$$\text{Min } u_t K_t + w_t L_t + e_t E_t$$

$$\overline{Q}_t = F(K_t, L_t, E_t)$$

where $\overline{Q}_t$ is the capacity added by the new vintage of equipment and F is either the double CES function or the CES + Cobb-Douglas function. The usual Lagrange calculation then gives the optimal factor quantities or, in the case of the double CES function:

$$\text{(B1)}\quad (\frac{K}{\overline{Q}})^*_t = Q_0^{-1} e^{-rT} . \left[(a_K^{\sigma} + a_E^{\sigma} (\frac{e}{u})_t^{1-\sigma})^{\frac{\sigma_1-1}{\sigma-1}} + a_L^{\sigma_1} (\frac{w}{u})_t^{1-\sigma_1}\right]^{\frac{\sigma_1}{1-\sigma_1}} . a_K^{\sigma} . \left[a_K^{\sigma} + a_E^{\sigma} (\frac{e}{u})_t^{1-\sigma}\right]^{\frac{\sigma_1-\sigma}{\sigma-1}}$$

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(1) In addition, when doing the estimation we replaced $\dot{w}$ by $\dot{p}$ (the expected rate of growth of the output price) in the calculation of u and u_e, which - perhaps - has the effect of making the relative prices w/p and e/p play a part in the model.

$$(B2)\quad \left(\frac{E}{\bar{Q}}\right)^{*}_{t} = Q_0^{-1}\, e^{-rT} \,.\, \left[\left(a_K^{\sigma} \left(\frac{u}{e}\right)_t^{1-\sigma} + a_E^{\sigma}\right)^{\frac{\sigma_1-1}{\sigma-1}} + a_L^{\sigma_1} \left(\frac{w}{e}\right)_t^{1-\sigma_1}\right]^{\frac{\sigma_1}{1-\sigma_1}} .\ a_E^{\sigma} .\ \left[a_K^{\sigma} \left(\frac{u}{e}\right)_t^{1-\sigma} + a_E^{\sigma}\right]^{\frac{\sigma_1-\sigma}{\sigma-1}}$$

$$(B3)\quad \left(\frac{L}{\bar{Q}}\right)^{*} = Q_0^{-1}\, e^{-rT}\, a_L^{\sigma_1} \,.\, \left[\left(a_K^{\sigma}\, u_t^{1-\sigma} + a_E^{\sigma}\, e^{1-\sigma}\right)^{\frac{\sigma_1-1}{\sigma-1}} + a_L^{\sigma_1}\, w^{1-\sigma_1}\right]^{\frac{\sigma_1}{1-\sigma_1}} .\ w^{-\sigma_1}$$

where u = cost of capital use

e = energy price

w = labour cost

$\bar{Q}$ = market constraint.

4. It will then be useful to obtain the ratios for the optimal factor quantities:

$$(B4)\quad \left(\frac{E}{K}\right)^{*}_{t} = \left(\frac{a_E}{a_K}\right)^{\sigma} \left(\frac{u}{e}\right)^{\sigma}_{t}$$

$$(B5)\quad \left(\frac{Z}{L}\right)^{*}_{t} = a_L^{-\sigma_1} \left(\frac{w}{P_{KE}}\right)^{\sigma_1}_{t} \quad \text{where}$$

$$(B6)\quad P_{KE} = \left(a_K^{\sigma}\, u^{1-\sigma} + a_E^{\sigma}\, e^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

With the CD + CES production function the expressions for the optimal factor quantities are simpler because we obtain:

$$(C1)\quad \left(\frac{K}{Q}\right)^{*}_{t} = Q_0^{-1}\, e^{-rT} \left(\frac{a}{1-a}\right)^{1-a} a_K^{\sigma} \left(\frac{w}{u}\right)^{1-a}_{t} \left[a_K^{\sigma} + a_E^{\sigma} \left(\frac{e}{u}\right)^{1-\sigma}_{t}\right]^{\frac{a}{1-\sigma} - 1}$$

$$(C2)\quad \left(\frac{E}{Q}\right)^{*}_{t} = Q_0^{-1}\, e^{-rT} \left(\frac{a}{1-a}\right)^{1-a} a_E^{\sigma} \left(\frac{w}{e}\right)^{1-a}_{t} \left[a_K^{\sigma} \left(\frac{u}{e}\right)^{1-\sigma}_{t} + a_E^{\sigma}\right]^{\frac{a}{1-\sigma} - 1}$$

$$(C3)\quad \left(\frac{L}{Q}\right)^{*}_{t} = Q_0^{-1}\, e^{-rT} \left(\frac{1-a}{a}\right)^{a} \left(\frac{u}{w}\right)^{a}_{t} \left[a_K^{\sigma} + a_E^{\sigma} \left(\frac{e}{u}\right)^{1-\sigma}_{t}\right]^{\frac{a}{1-\sigma}}$$

with

$$(C4)\quad \left(\frac{E}{K}\right)^{*}_{t} = \left(\frac{a_E}{a_K}\right)^{\sigma} \left(\frac{u}{e}\right)^{\sigma}_{t}]$$

ANNEX 2

Derivation of the expression for the scrapping rate

1. Starting from an equilibrium growth regime where output and the relative output price of the variable factors grow at the rate g (therefore the volume of these factors is constant, relaxing this assumption does not change the results), capacity installed at time t is $Q_{t-T}(1 + g)^t$; scrapped capacity is Q_{t-T}. The scrapping rate is thus equal to:

$$\delta = \frac{Q_{t-T}}{\sum_{\theta=0}^{T} Q_{t-T}(1+g)^{\theta}} = \frac{-g}{1-(1+g)^{T+1}} \simeq \frac{1}{T} - g/2$$

In this regime the share of variable costs in output is constant for all new vintages installed and for all vintages as a whole (though at two different constant levels, of course). Firstly, assume that a piece of equipment is scrapped when variable costs exceed the output value that can be produced with it. T is therefore such that:

$(CV)_{t-T}/pQ_{t-T} = 1$ where CV = variable cost
p = output cost

Using the expression for δ and multiplying top and bottom by T and taking the logarithms, gives:

$$T \simeq - z/g \log (P_V V/pQ)$$

where P_V = (weighted) price of variable inputs;

V = volume of variable inputs

Q = total output

so that $P_V V/pQ$ = (constant) share of variable costs in output.

This gives in the equilibrium regime:

$$\delta = \frac{1}{T} - \frac{g}{2} = \frac{g}{2} \left[-1 - \frac{1}{\log \left(\frac{P_V V}{pQ}\right)}\right]$$

2. Assume now that, in a particular year, the weighted price (relative to the output price) of the variable inputs increases by g + μ, rather than g. This will have two effects:

- in the short term, additional equipment that has become non-profitable is scrapped corresponding to μ/g year of investment. If the scrapping rate in the equilibrium growth regime is δ_0, in the short term it becomes:

$$\delta_1 = \delta_0 + \frac{\mu/g}{T} \simeq \delta_0 \left(1 + \frac{\mu}{g}\right) + \frac{\mu}{2}$$

- in the long term δ_0 becomes δ_2 corresponding with the new higher value of the share of costs P_vV/pQ.

Noting that μ is the rate of growth of P_vV/pQ:

$$\delta = \frac{g}{2}\left[-1 - \frac{1}{\log\left(\frac{P_vV}{PQ}\right)}\right]\left(1 + \left(\frac{\dot{P}_vV}{PQ}\right)/g\right) + \left(\frac{P_vV}{PQ}\right)/2$$

In fact the above expression was not used in the estimation of δ because of its complexity and the problem of quantifying g. It was linearised (and the logarithm) to give:

$$\delta = A + \frac{B}{1-\frac{P_vV}{PQ}} + C\left(\frac{\dot{P}_vV}{PQ}\right) \quad B,\ C > 0.$$

ANNEX 3

Model determining employment and hours worked

1. The firm pays W_o for one hour worked when employees are working normal hours $(H = H_o)$, W_1 for one hour of overtime $(H_o \leqslant H \leqslant H\ max)$, λW_o for one hour not worked when worktime declines to less than normal hours (or the hours lying between H and H_o when $H\ min \leqslant H \leqslant H_o$, hours worked then being paid at the rate W_o). Let $W_1 = \theta W_o$ $(\theta \geqslant 1$, while $0 \leqslant \lambda \leqslant 1)$. The productivity of one hour of overtime is equal to k times $(0 \leqslant k \leqslant 1)$ hourly productivity for $H \leqslant H_o$. The problem of shift working is ignored here. Employment cannot therefore exceed E max, which is the number of jobs established. W_2 is the cost associated with employment adjustment(1). The firm's programme is therefore:

$$\min_{E,H} W_o H_{min} E + (\theta-1)W_o E \max (H - H_o, 0) + \lambda W_o E \max (H_o - H, 0)$$

$$+ W_o E (H - H_{min}) + W_2 H_o \left(\frac{E - E_{-1}}{E_{-1}}\right)^2$$

$$= W_o HE + (\theta-1)W_o E \max (H - H_o, 0) + \lambda W_o E \max (H_o -$$

$$+ W_2 H_o \left(\frac{E - E_{-1}}{E_{-1}}\right)^2$$

Subject to the constraints:

$$Q = Q_o [EH - (1 - k)E \max (H - H_o, 0)]$$

$$H_{min} \leqslant H \leqslant H_{max} \quad 0 \leqslant E \leqslant E_{max}$$

where Q = output

E = employment

If the limits H_{min}, H_{max} and E_{max} are sufficiently distant and are not reached, the result is:

$$\text{If } H < H_o \quad H^- = H_o \frac{Q}{Q_o E_{-1} H_o} \frac{1}{1 - \frac{W_o}{2W_2}\lambda}$$

---

(1) To obtain an optimum it has to be assumed that the adjustment cost is an increasing function (in this instance quadratic) of the number of persons hired or laid off.

$$\text{If } H > H_o \quad H^+ = H_o \left[\frac{Q}{Q_o E_{-1} H_o} \; \frac{1}{k} \; \frac{1}{1 + \frac{W_o}{2W_2} \left(\frac{Q}{k} - 1\right)} - \frac{1 - k}{k} \right]$$

$$\text{If } 1 - \frac{W_o}{2W_2}\lambda < \frac{Q}{Q_o E_{-1} H_o} < 1 + \frac{W_o}{2W_2} \left(\frac{Q}{k} - 1\right), \; H = H_o$$

For small fluctuations in output, the employment adjustment cost is smaller than the adjustment cost for overtime or short-time working. When the change in output exceeds these limits, the adjustment comes from hours worked. If demand is strong, hours worked are greater than normal hours and the bigger W_2/W_o, the bigger the increase in hours worked; if demand is weak, hours worked are less than normal hours and the bigger W_2/W_o, the steeper the fall in hours worked. For small fluctuations in output, hours worked are equal to normal hours H_o. H^- is an increasing function of λ and $W_o W_2$ (hours worked deviate little from H_o when short time is well-paid and when the employment adjustment cost is small); H^+ is a decreasing function of 0 and W_o/W_2; the sign of the derivative of H^+ with respect to k is ambiguous because the low productivity of overtime encourages the firm to make little use of it, but on the other hand results in a greater number of hours being worked to obtain the same output.

2. To arrive at the estimated form of the hours worked equation, which is a function of observed magnitudes, we now have to write the expression for:

$$\frac{Q}{Q_o E_{-1} H_o}$$

If, in the previous period, hours worked were equal to normal hours H_o, then:

$$Q_o E_{-1} H_o = Q_{-1} \text{ and } \frac{Q}{Q_o E_{-1} H_o} = 1 + \dot{Q}$$

($\dot{Q}$ = rate of growth of output).

If, in the previous period, H differed from H_o, then when $H < H_o$:

$$Q_o E_{-1} H_{-1} = Q_{-1}$$

$$\frac{H_o}{H_{-1}} = \frac{Q_o E_{-2} H_o}{Q_{-1}} \left(1 - \left(\frac{W_o}{2W_2}\right)_{-1} \lambda\right)$$

and if at t-2, $H_{-2} = H_o$

$$\frac{Q}{Q_oE_{-1}H_o} = (1 + \dot{Q})(1 + \dot{Q}_{-1})\left[1 - \left(\frac{W_o}{2W_2}\right)_{-1}\right]^{-1}$$

To be rigorous we ought therefore to introduce W_2/W_o with lags, but because of the collinearities with the point value of that ratio this was impossible.

If $H > H_o$, we arrive at an identical solution except in the case where, in the previous period, very heavy demand drew employment toward maximum employment, subject to the capacity constraint. In this case:

$$\frac{Q}{Q_oE_{-1}H_o} = \frac{Q}{Q_oE_{max}H_o} = \frac{Q}{\text{capacity with normal hours}} = Tut$$

where Tut = rate of capacity utilisation with normal hours, which was calculated after the investment equations were estimated.

ANNEX 4

Share of non-wage cost in total labour cost /after deduction of employers' social insurance contributions (%)7

	United States	Germany	France	United Kingdom	Japan	Canada
1968	9.0	16.3	15.6	6.7	6.5	10.4
1969	9.6	16.3	14.5	7.5	5.6	10.7
1970	10.8	17.1	15.3	7.9	5.0	11.0
1971	10.3	17.9	15.8	8.5	5.1	11.0
1972	10.1	18.1	16.2	7.8	5.3	10.5
1973	11.2	19.8	17.1	7.2	4.8	10.4
1974	10.4	20.7	18.2	7.9	4.7	11.3
1975	10.7	21.4	18.4	8.1	4.6	12.0
1976	10.6	21.5	17.8	6.5	4.6	11.5
1977	10.5	21.8	17.5	7.2	4.7	12.9
1978	10.8	22.4	17.1	7.4	5.4	14.4
1979	12.6	22.5	16.9	10.5	6.7	14.6
Average	10.5	19.6	16.7	7.8	5.2	11.7

ANNEX 5

Capacity utilisation rate series

Capacity utilisation rate (%)

		United States			Germany			France			United Kingdom			Japan			Canada		
		I	II	III	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III
1964	1	95.5	96.6	84.6	96.1	98.6	87.6												
	2	95.7	98.0		96.8	99.9	87.8												
1965	1	96.4	101.3	88.9	97.6	102.4	88.2				96.8	99.8	44						
	2	96.2	103.1		96.9	100.9	87.7				94.9	98.9	50				86.5	101.5	84.4
1966	1	95.9	105.9	91.2	96.1	100.7	86.8				93.4	99.4	50				89.7	102.5	90.2
	2	94.2	106.2		92.0	94.9	84.2				92.2	96.5	44				90.0	100.8	87.9
1967	1	91.5	102.0	87.6	89.1	88.4	77.7				91.3	95.3	32				91.4	98.2	86.0
	2	91.0	102.1		89.9	92.2	79.8				91.2	95.2	30				91.4	96.6	85.2
1968	1	92.0	103.7	86.8	91.8	92.8	83.2	93.0	92.7	84.2	92.7	98.3	39				93.7	96.4	84.9
	2	92.1	103.4		95.5	98.0	86.3	95.5	101.2	87.1	92.6	98.1	49				97.0	98.3	87.2
1969	1	91.5	103.3	85.0	97.4	100.5	89.1	94.7	100.2	85.7	91.3	99.2	54				98.1	99.1	88.7
	2	89.9	101.5		99.9	102.9	90.0	95.5	98.5	86.3	90.1	98.3	51				99.3	97.3	87.3
1970	1	87.1	95.1	77.4	98.7	103.4	92.1	96.3	100.2	86.1	89.0	96.6	47				98.6	93.8	84.6
	2	85.6	90.9		98.0	101.9	90.0	96.0	99.9	85.5	88.3	96.5	49				97.1	89.5	81.2
1971	1	86.0	90.9	76.3	95.1	101.2	87.9	95.4	99.9	85.7	88.0	94.4	36	89.5	99.0	123.9	98.6	91.4	82.3
	2	86.7	91.2		93.3	97.4	85.6	95.4	100.5	85.8	87.3	92.7	26	88.4	95.1	122.3	101.3	93.1	85.0
1972	1	88.9	94.7	81.0	92.6	97.7	84.6	95.3	100.4	85.9	86.1	91.9	29	90.0	93.4	122.0	101.9	93.3	85.3
	2	91.0	98.4		92.5	96.7	85.5	95.0	100.4	86.6	87.7	95.3	38	90.9	95.0	124.6	102.6	94.5	88.1
1973	1	93.6	102.1	85.0	93.5	99.1	87.4	95.4	101.5	87.4	91.8	99.5	49	94.0	99.8	129.3	105.0	97.3	91.5
	2	92.3	101.0		92.0	98.2	86.9	94.4	100.8	87.6	91.7	99.7	57	91.4	100.4	126.9	105.1	97.3	92.0
1974	1	89.4	97.9	81.5	90.8	99.8	84.0	94.2	102.5	87.4	88.2	98.9	38	86.9	97.2	122.2	103.3	100.2	92.9
	2	86.4	92.9		88.3	94.7	81.0	92.3	97.0	84.4	85.3	97.5	42	86.2	86.1	111.9	98.1	95.9	88.5
1975	1	83.0	81.8	70.7	85.0	89.0	75.9	89.9	89.0	79.0	83.6	91.1	33	85.0	75.4	97.8	94.5	90.5	82.0
	2	85.7	88.5	74.3	86.7	87.8	76.2	90.1	87.9	78.3	84.0	86.6	24	85.3	75.7	101.7	93.3	87.9	81.5
1976	1	88.6	91.9	77.5	90.2	91.2	79.3	91.4	91.3	81.8	86.5	86.9	21	87.0	80.0	108.3	94.9	90.3	83.8
	2	89.6	92.2	78.6	91.6	92.0	81.2	92.1	90.5	83.7	87.7	87.3	29	87.5	81.6	108.6	92.6	88.3	83.2
1977	1	92.1	93.5	80.3	91.9	92.6	80.9	91.7	88.3	83.6	86.8	86.2	33	88.8	81.2	108.7	91.5	88.7	82.9
	2	93.7	95.0	81.3	91.9	91.1	80.0	90.3	84.3	83.2	87.5	84.2	31	88.8	80.0	105.6	90.4	87.5	81.9
1978	1	93.7		81.8	92.6		80.6	90.8		83.5	89.3		33	89.5		109.1	89.6	—	83.0
	2	94.0		84.1	93.2		81.9	90.8		83.8	89.8		36	89.7		111.9	89.4		85.7
1979	1	93.3		85.2	94.3		84.2	90.8		84.4	89.4		41	89.8		116.8	87.4		86.6
	2	91.7		83.3	94.5		85.3	92.1		84.8	87.7		43	90.3		119.7	84.9		85.8
1980	1	89.1		80.6				90.4		85.5	84.7		33				78.7		81.7
	2	87.4		77.3				88.2		84.2	82.8		20				74.9		80.0

I = Our estimate. II = IMF estimate. III = National sources.

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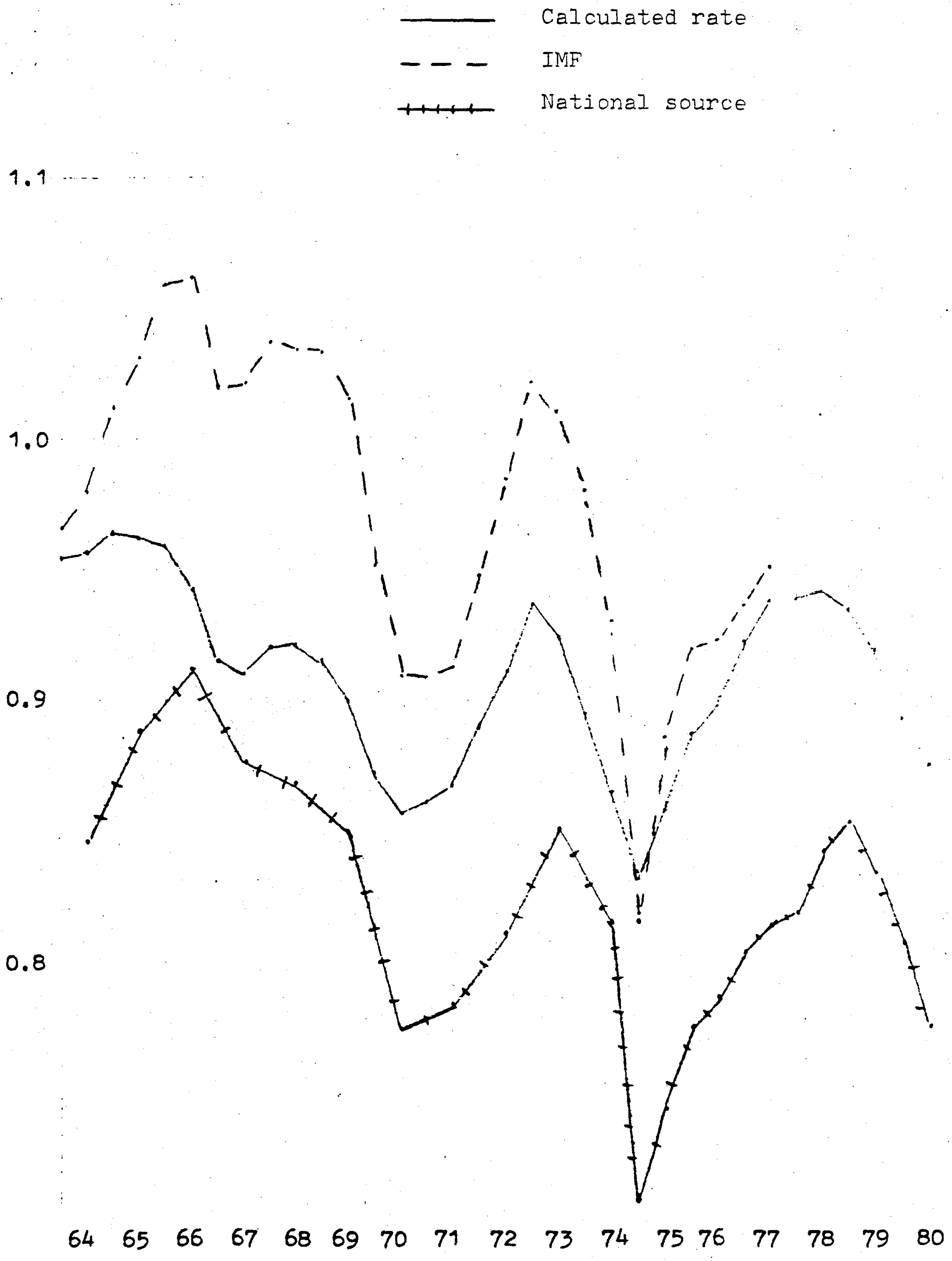

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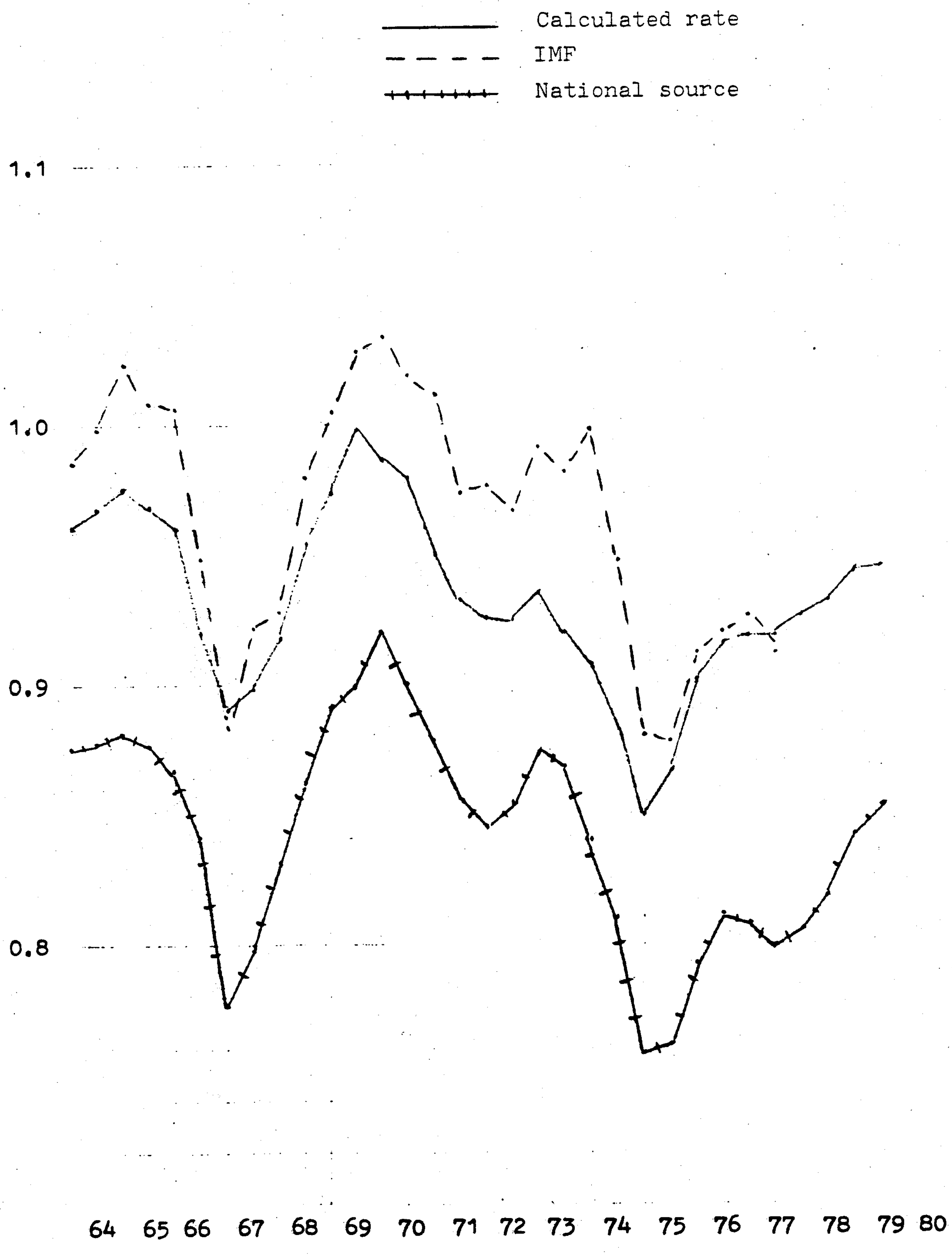
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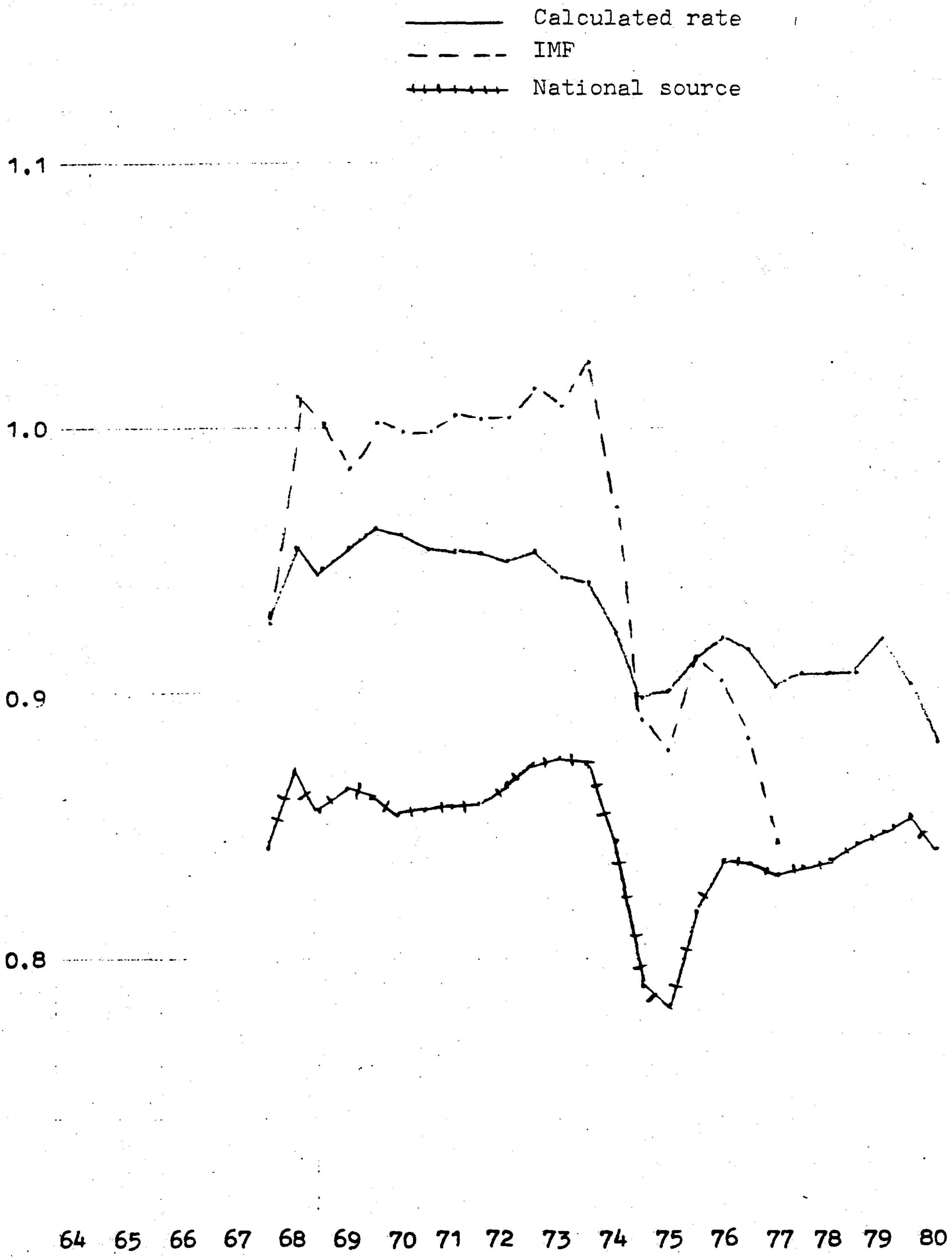
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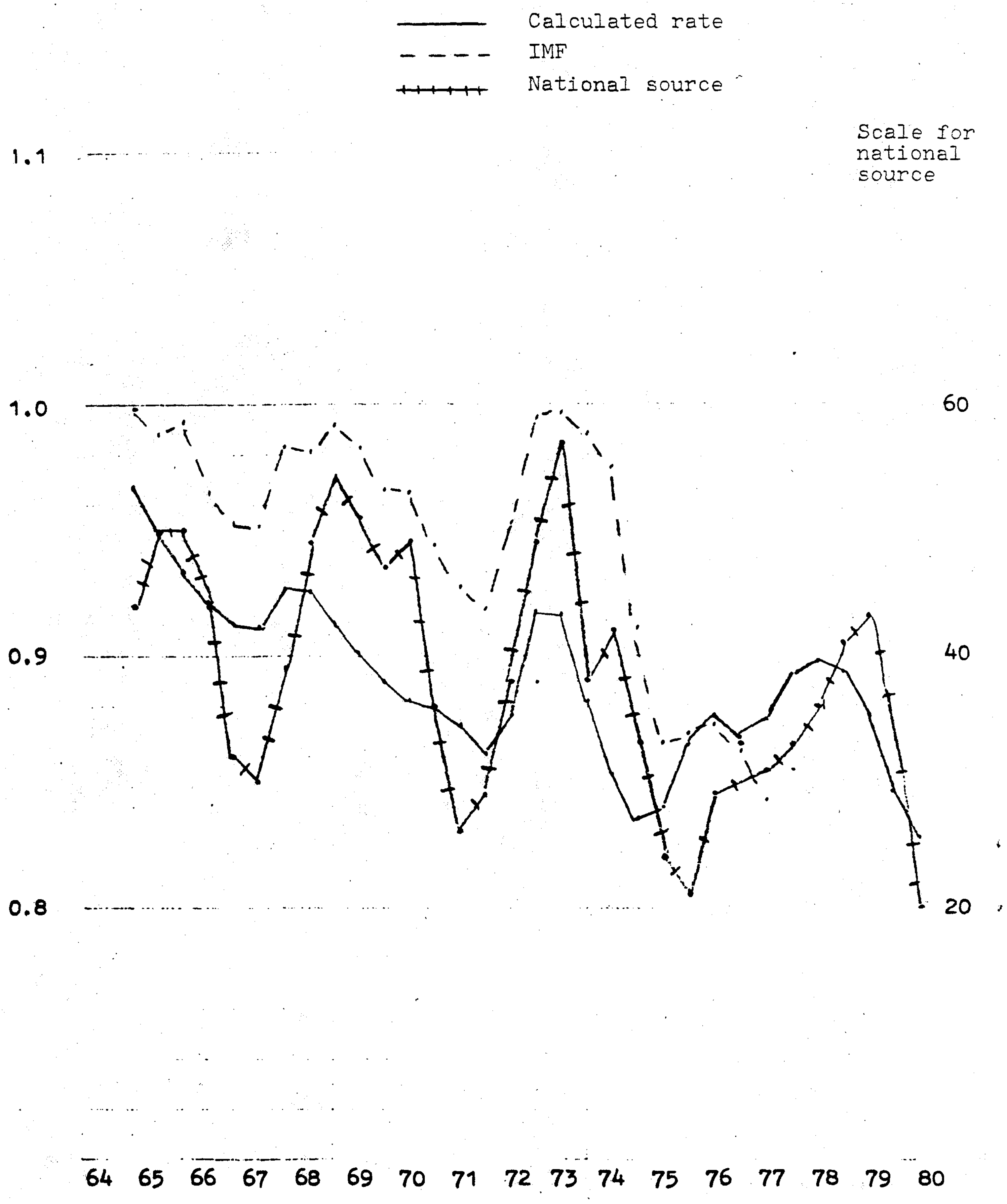

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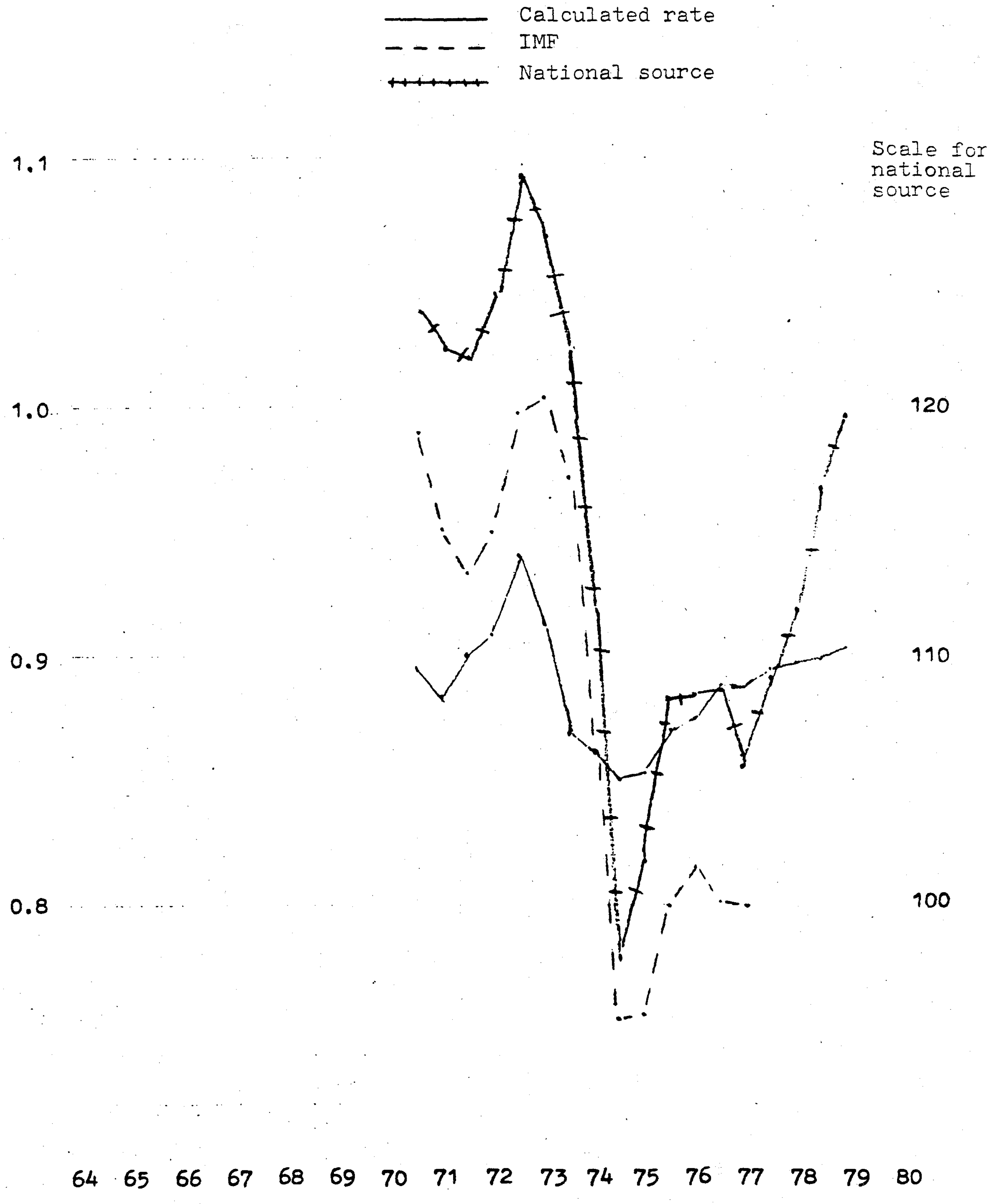
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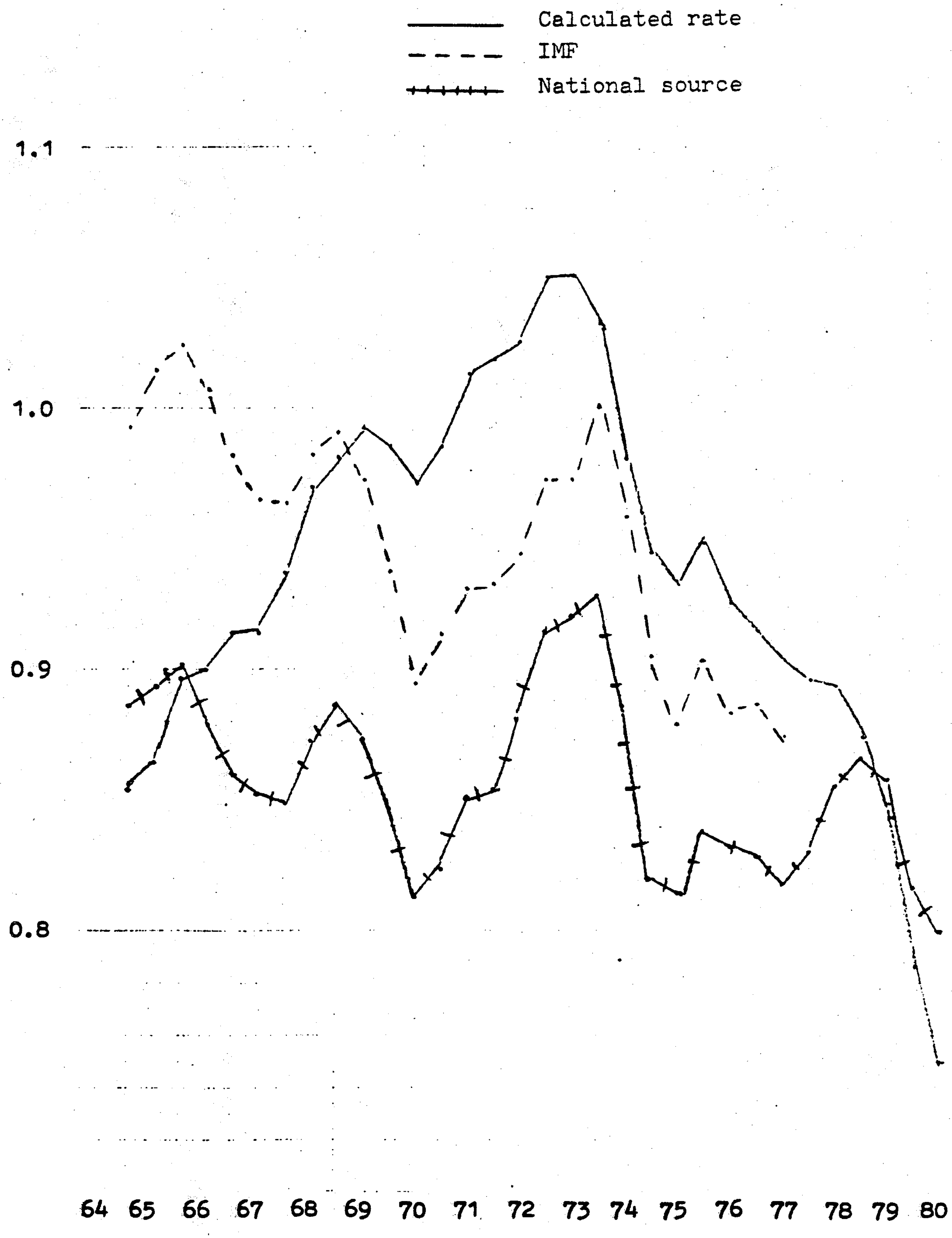
CANADA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